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THE STABILITY OF ECCENTRICALLY STIFFENED CIRCULAR CYLINDERS

VOLUME III

BUCKLING OF LONGITUDINALLY STIFFENED CYLINDERS; AXIAL COMPRESSION

GENERAL DYNAMICS
Convair Division

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THE STABILITY OF ECCENTRICALLY STIFFENED CIRCULAR CYLINDERS.

VOLUME III:

BUCKLING OF LONGITUDINALLY STIFFENED CYLINDERS; AXIAL COMPRESSION

Prepared for the
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GDC-DDG-67-006

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ABSTRACT

This is the third of six volumes, each bearing the same report number, but dealing with separate problem areas concerning the stability of eccentrically stiffened circular cylinders. The complete set of documents was prepared under NASA Contract NAS8-11181. This particular volume deals with the buckling of longitudinally stiffened circular cylinders under axial compression. Analysis methods are presented in the forms of procedures, curves, and digital computer programs. These methods apply equally well to cylinders which incorporate only longitudinal stiffening (stringers) and to sections which lie between rings in cylinders having both axial and circumferential stiffening. Application to the latter case is valid only when the critical load for general instability exceeds the critical load for the so-called panel instability mode. Since the contents of this volume are based upon a Donnell-type small-deflection theory, the proposed methods should be used in conjunction with empirical knock-down factors to account for the effects of initial imperfections. In addition, the Donnell assumptions preclude application to non-axisymmetric buckle patterns where the number of circumferential waves is small.

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DEFINITION OF SYMBOLS

Symbol	Definition
A_s	Area of a single stringer (no skin included).
$A_{11}, A_{22}, A_{12}, A_{33}$	Elastic constants [see equations (2-4) and Table X].
a	Ring spacing.
b	Stringer spacing.
b_s	Thickness of integral longitudinal stiffener (see Table X).
C_F	Fixity factor.
$C_{11}, C_{22}, C_{12}, C_{21}$	Eccentricity coupling constants [see equations (2-4) and Table X].
$D_{11}, D_{22}, D_{12}, D_{33}$	Elastic constants [see equations (2-4) and Table X].
E	Young's modulus.
E_{tan}	Tangent modulus in compression.
F	Parameter defined by equation (2-7).
G	Modulus of elasticity in shear.
G_{tan}	Tangent modulus in shear.
H	Parameter defined by equation (2-6).
h	Corrugation pitch ÷ 4; (see notes of Table X).
h_s	Depth of integral longitudinal stiffener (see Table X).
I	Moment of inertia.

DEFINITION OF SYMBOLS
(Continued)

Symbol	Definition
$\overline{I}_x$	Running centroidal moment of inertia of shell wall cross section lying in plane normal to axis of revolution (see notes of Table X).
$\overline{I}_y$	Running centroidal moment of inertia of shell wall cross section lying in radial plane [$= t^3/12$ for longitudinally stiffened cylinders].
L	Overall length of cylinder.
L′	Effective length ($= \frac{L}{m}$).
m	Number of axial half-waves in buckle pattern.
m_L	Value defined by equation (4-4).
N*	Minimization factor defined by equation (2-22).
$\left(\overline{N}_{THIEL}\right)$	Loading parameter defined in equations (2-3), (positive for tensile loading).
$\left(\overline{N}_{THIEL}\right)_c$	$= -\overline{N}_{THIEL}$, (positive for compressive loading).
$\overline{N}_x$	Applied longitudinal tensile running load acting at the centroid of the effective skin-stringer combination.
$\left(\overline{N}_x\right)_c$	Applied longitudinal compressive running load acting at the centroid of the effective skin-stringer combination ($= -\overline{N}_x$).
$\left[\left(\overline{N}_x\right)_c\right]_{cr}$	Critical value of applied longitudinal compressive running load acting at the centroid of the effective skin-stringer combination.

DEFINITION OF SYMBOLS
(Continued)

Symbol	Definition
n	Number of circumferential full-waves in buckle pattern; Ramberg-Osgood material parameter.
R	Radius of middle surface of basic cylindrical skin.
t	Thickness of basic cylindrical skin.
t_c	Skin thickness of corrugated wall.
t_{eff}	Equivalent thickness used in the computation of the knock-down factor Γ (see Volume V).
$\bar{t}_x$	Thickness of appropriate smeared-out area of cross section lying in plane normal to axis of revolution. [See Table X]
u, v, w	Reference - surface displacements (see Figure 2).
x, y, z	Coordinates (see Figure 2).
Z	Parameter defined by equation (2-2).
$\bar{z}_x$	Eccentricity (see Table X).
α	Parameter defined in equations (2-3).
β	Parameter defined in equations (2-3).
Γ	Knock-down factor which accounts for effects of initial imperfections. (see Volume V).
γ	Parameter defined in equations (2-3).
Δ_x	Deflection defined in note (h) of Table X.
$\Delta\theta$	Rotation defined in note (i) of Table X.

DEFINITION OF SYMBOLS
(Continued)

Symbol	Definition
δ_x	Deflection defined in note (h) of Table X.
$\delta\theta$	Rotation defined in note (i) of Table X.
η_p	Parameter defined in equations (2-3).
η_s	Parameter defined in equations (2-3).
ν	Poisson's ratio.
ρ_x	Local centroidal radius of gyration for shell wall cross section lying in a plane which is normal to the axis of revolution. [see equation (5-2)].
(Σd_i)	Total peripheral length of corrugation center-line (see Table X and its notes).
σ_{cc}	Crippling stress.
σ_{cr}	Critical compressive stress.
σ_{cy}	Compressive yield stress.
$\sigma_{.7}$	Ramberg-Osgood material parameter.

SECTION 1

INTRODUCTION

The contents of this volume deal with the buckling of longitudinally stiffened circular cylinders which are subjected to axial compression. Analysis methods are presented in the form of procedures, curves, and digital computer programs. These methods apply equally well to cylinders which incorporate only longitudinal stiffening (stringers) and to sections which lie between rings in cylinders having both axial and circumferential stiffening. Application to the latter case is only valid when the critical load for general instability (see GLOSSARY, Volume I [1]) exceeds the critical load for the so-called panel instability (see GLOSSARY, Volume I [1]) mode. That is, in all applications it is assumed that, during buckling, the ends of the longitudinally stiffened sections are fully restrained against radial displacement. Various degrees of end rotational restraint are considered by means of an engineering approximation. Since the primary theoretical foundations for this volume lie in small-deflection shell theory, it is recommended that the proposed analysis methods be used in conjunction with empirical knock-down factors to account for the effects of initial imperfections. Appropriate criteria for these factors are given in Volume V [2]. Since most practical stiffened cylinders are "effectively thick", in general their related reductions will not be nearly so severe as those encountered for thin-walled isotropic cylinders. It is also important to note that the basic orthotropic shell equation of this volume is based upon Donnell-type simplifications [3]. As a result, the methods presented here cannot be applied when the instability manifests itself in a non-axisymmetric buckle pattern which has a small number of circumferential waves. The rule-of-thumb guideline is offered here that these methods should be considered invalid for cases where

$$0 < n < 2 \qquad (1\text{-}1)$$

Numbers in brackets [ ] in the text denote references listed in SECTION 8.

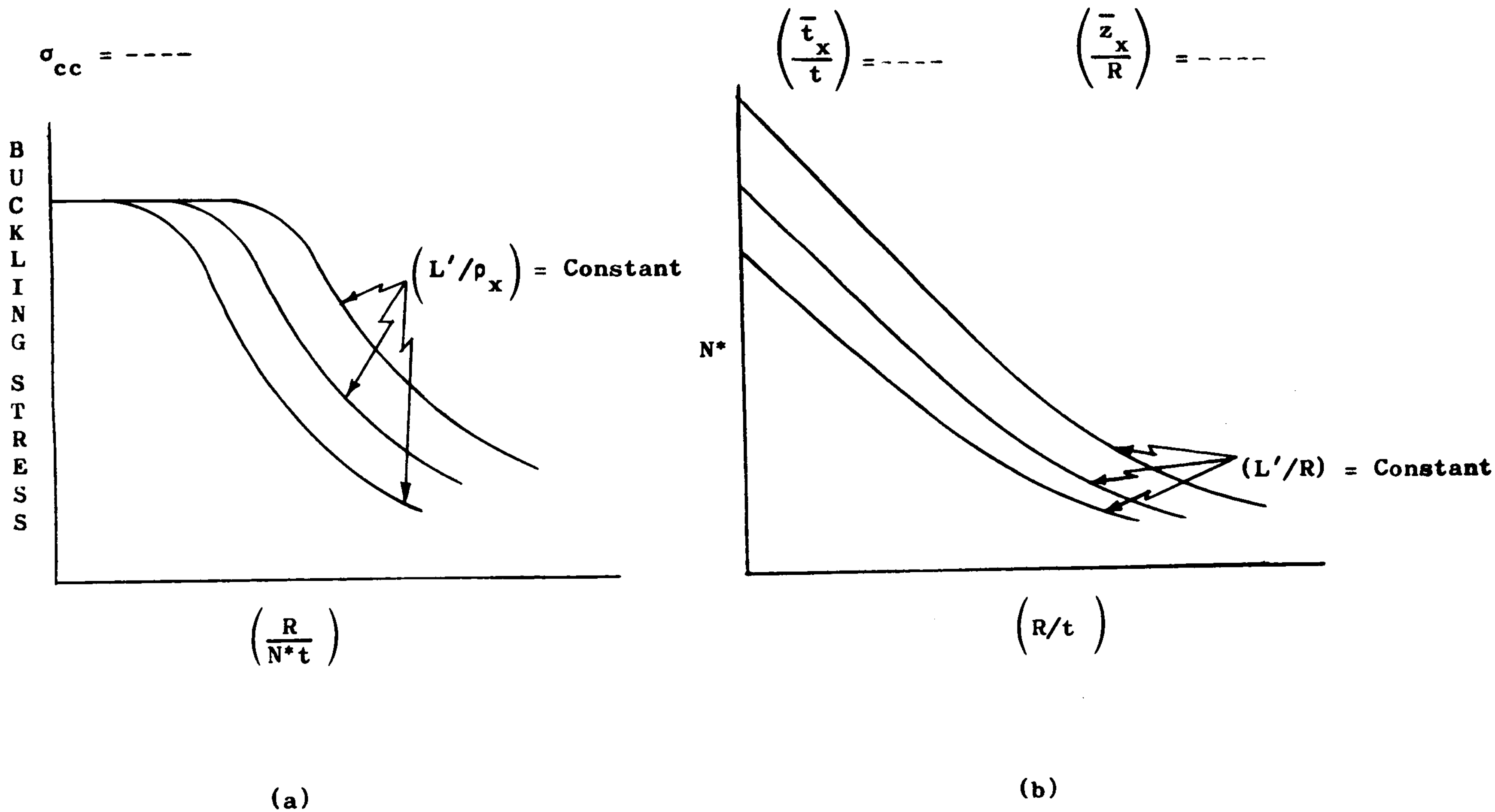


Figure 1 - Schematic Representation of Primary Buckling Curves for Longitudinally Stiffened Cylinders

The primary buckling curves of this volume are of two basically different forms. These are depicted in Figure 1. In order to arrive at the predicted buckling stresses, N* values must first be found from the curves of Figure 1(b). One may then enter the curves of Figure 1(a) to obtain the desired buckling stresses. This type of format evolved as an expediency which was consistent with the scope of work under NASA Contract NAS8-11181. The two separate digital computer programs which were used to generate these curves could now be combined into one program. This could then be followed by a consolidation of the indicated two-step analysis process into a single-step operation which involved only one type of plotting format. This further improvement is included among the recommendations of reference 4.

SECTION 2

EQUATIONS

The following orthotropic cylinder equation provides the basis for the methods given in this volume:

$$\left(\bar{N}_{THIEL}\right)_c = \frac{\left[1 + 2\eta_p\sqrt{\gamma}\,\beta^2 + \gamma\,\beta^4\right]}{4\alpha\beta^4} + \frac{\alpha\,\beta^4\,(Z)^2}{\left[1 + 2\eta_s\beta^2 + \beta^4\right]} \tag{2-1}$$

where,

$$Z = \left[1 - \frac{C_{11} + C_{22}}{2\alpha\,(A_{22}D_{22})^{1/2}\beta^2} - \frac{C_{12}}{2\alpha\,A_{22}(D_{22}/A_{11})^{1/2}} - \frac{C_{21}}{2\,\alpha\,(A_{11}D_{22})^{1/2}\beta^4}\right] \tag{2-2}$$

A detailed derivation of these relationships is given in reference 5 where the coordinate system shown in Figure 2 was used. Some general background

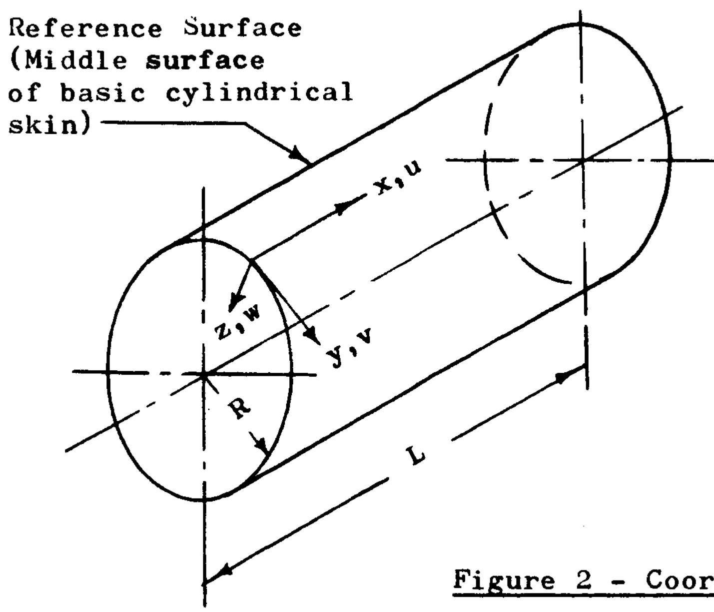


Figure 2 - Coordinate System

information concerning equations (2-1) and (2-2) is given in Volume I [1]. As noted there, these equations have been written in rather compact, instructive forms through the introduction of the following parameters, most of which were first proposed by Thielemann [6]:

$$\left(\bar{N}_{THIEL}\right)_c = - \bar{N}_{THIEL} = - \frac{\bar{N}_x R}{2}\left(\frac{A_{11}}{D_{22}}\right)^{1/2}$$

$$\eta_s = \frac{\left(A_{12} + \frac{A_{33}}{2}\right)}{\sqrt{A_{11}A_{22}}}$$

$$\eta_p = \frac{D_{12} + 2D_{33}}{\sqrt{D_{11}D_{22}}}$$

$$\gamma = \frac{D_{11}A_{11}}{D_{22}A_{22}} \qquad (2\text{-}3)$$

$$\beta = \left(\frac{m}{n}\right)\left(\frac{\pi R}{L}\right)\left(\frac{A_{22}}{A_{11}}\right)^{1/4}$$

$$\alpha = \frac{L^2}{2Rm^2\pi^2 A_{22}\left(\frac{D_{22}}{A_{11}}\right)^{1/2}}$$

The various A_{ij}'s, D_{ij}'s, and C_{ij}'s of equations (2-2) and (2-3) are very important fundamental constants. The physical significance of these constants is discussed in Volume I [1]. The A_{ij}'s and D_{ij}'s are usually referred to as elastic constants while the C_{ij}'s might be identified as eccentricity coupling constants.

Equations (2-1) and (2-2) can be specialized to the case of a cylinder having only longitudinal stiffening by using the following simplified expressions for the elastic constants and the eccentricity coupling constants:

$$A_{11} = \frac{1}{E\bar{t}_x} \qquad A_{22} = \frac{1}{Et}$$

$$A_{33} = \frac{1}{Gt} \qquad A_{12} = A_{21} = \frac{-\nu}{E\bar{t}_x}$$

$$D_{11} = E\bar{I}_x \qquad D_{22} = \frac{E\bar{I}_y}{(1-\nu^2)} = \frac{Et^3}{12(1-\nu^2)} \tag{2-4}$$

$$D_{33} = \frac{Gt^3}{12} \qquad D_{12} = D_{21} = \frac{\nu E\bar{I}_y}{(1-\nu^2)} = \frac{\nu Et^3}{12(1-\nu^2)}$$

$$C_{11} = \bar{z}_x \qquad C_{22} = 0$$

$$C_{12} = -\nu\bar{z}_x \qquad C_{21} = 0$$

The notation used in these formulas is fully clarified in the DEFINITION OF SYMBOLS. The derivations of these relationships are based on the assumption that the ratio $(\bar{I}_y/\bar{I}_x)$ is small compared to unity. That is, it is assumed that the bending stiffness of the skin is small with respect to the bending stiffness of the appropriate skin-stringer combination. Most practical longitudinally stiffened cylinders will comply with this condition. In general, departures will only occur where the stringers are very shallow. In such cases one must use more refined expressions than are furnished in equations (2-4).

The substitution of $C_{22} = C_{21} = 0$ into equations (2-1) and (2-2) gives the following result:

$$\left(\bar{N}_{THIEL}\right)_c = \frac{\left[1 + 2\eta_p\sqrt{\gamma}\,\beta^2 + \gamma\beta^4\right]}{4\alpha\beta^4} + \frac{\alpha\beta^4\left[1 - \dfrac{C_{11}}{2\alpha(A_{22}D_{22})^{1/2}\,\beta^2} - \dfrac{C_{12}}{2\alpha A_{22}(D_{22}/A_{11})^{1/2}}\right]^2}{\left[1+2\eta_s\beta^2 + \beta^4\right]} \tag{2-5}$$

It is noted that, except for the C_{12} term, this equation is identical to equation (36) of reference 7 where the C_{12} term was evidently discarded as a negligible quantity for the particular test specimens of interest there. For convenience, the following quantities are now defined:

$$H = 1 - \frac{C_{12}}{2\alpha A_{22}(D_{22}/A_{11})^{1/2}} \tag{2-6}$$

$$F = \frac{C_{11}}{2\alpha(A_{22}D_{22})^{1/2}} \tag{2-7}$$

These expressions may be substituted into equation (2-5) to obtain

$$\left(\bar{N}_{THIEL}\right)_c = \frac{\left[1 + 2\eta_p\sqrt{\gamma}\,\beta^2 + \gamma\beta^4\right]}{4\alpha\beta^4} + \frac{\alpha\beta^4\left[H - \dfrac{F}{\beta^2}\right]^2}{\left[1 + 2\eta_s\beta^2 + \beta^4\right]} \tag{2-8}$$

For convenience this will now be rewritten as follows:

$$\left(\bar{N}_{THIEL}\right)_c = \frac{\Upsilon}{4\alpha} + \left\{ \frac{\eta_p\sqrt{\Upsilon}}{2\alpha\beta^2} + \frac{1}{4\alpha\beta^4} + \frac{\alpha\beta^4\left[H - \frac{F}{\beta^2}\right]^2}{\left[1 + 2\eta_s\beta^2 + \beta^4\right]} \right\} \tag{2-9}$$

From the first of equations (2-3), it is noted that

$$\left(\bar{N}_{THIEL}\right)_c = -\frac{R\bar{N}_x}{2}\sqrt{\frac{A_{11}}{D_{22}}} \tag{2-10}$$

Substitution of this equality into equation (2-9) gives the result

$$-\bar{N}_x = \frac{2}{R}\sqrt{\frac{D_{22}}{A_{11}}}\left(\frac{\Upsilon}{4\alpha}\right) + \frac{2}{R}\sqrt{\frac{D_{22}}{A_{11}}} \left\{ \frac{\eta_p\sqrt{\Upsilon}}{2\alpha\beta^2} + \frac{1}{4\alpha\beta^4} + \frac{\alpha\beta^4\left[H - \frac{F}{\beta^2}\right]^2}{\left[1 + 2\eta_s\beta^2 + \beta^4\right]} \right\} \tag{2-11}$$

where the loading $\bar{N}_x$ is positive in tension. It will therefore prove convenient to define

$$\left(\bar{N}_x\right)_c = -\bar{N}_x \tag{2-12}$$

where $\left(\bar{N}_x\right)_c$ is positive in compression. In addition, from equations (2-3) it is known that

$$\Upsilon = \frac{D_{11}A_{11}}{D_{22}A_{22}} \tag{2-13}$$

and

$$\alpha = \frac{L^2}{2Rm^2\pi^2A_{22}\left(\frac{D_{22}}{A_{11}}\right)^{1/2}} \tag{2-14}$$

By using equations (2-13) and (2-14), the following equality is easily obtained:

$$\frac{2}{R}\sqrt{\frac{D_{22}}{A_{11}}}\left(\frac{\Upsilon}{4\alpha}\right) = \frac{m^2\pi^2 D_{11}}{L^2} \tag{2-15}$$

In Volume I [1], it is pointed out that the quantity D_{11} constitutes the longitudinal flexural stiffness per unit length of circumference. From equations (2-4) it is seen that this elastic constant can be formulated as follows:

$$D_{11} = E\bar{I}_x \tag{2-16}$$

where

$\bar{I}_x$ = Shell wall (including effective skin and stringer) local moment of inertia per unit length of circumference taken about the centroidal axis of the effective skin-stringer combination.

By direct substitution and simplification, equations (2-4) lead to the following equality:

$$\frac{2}{R}\sqrt{\frac{D_{22}}{A_{11}}} = \frac{1}{\sqrt{3(1-\nu^2)}}\,\frac{Et}{R}\sqrt{t\bar{t}_x} \tag{2-17}$$

By inserting equations (2-12), (2-15), and (2-17) into equation (2-11), one may then obtain

$$\left(\bar{N}_x\right)_c = \frac{m^2\pi^2 D_{11}}{L^2} + \frac{\sqrt{t\bar{t}_x}}{\sqrt{3(1-\nu^2)}}\left(\frac{Et}{R}\right)\left\{\frac{\eta_p\sqrt{\Upsilon}}{2\alpha\beta^2} + \frac{1}{4\alpha\beta^4} + \frac{\alpha\beta^4\left[H - \dfrac{F}{\beta^2}\right]^2}{\left[1 + 2\eta_s\beta^2 + \beta^4\right]}\right\} \tag{2-18}$$

It should be observed that m (the number of longitudinal half-waves) appears in both the first and the bracketed terms of this equation. Its presence in the latter is due to the formulas for α, β, and F given by equations (2-3) and (2-7). Hence, for any particular selected m value, a corresponding critical axial loading $\left[\left(\bar{N}_x\right)_c\right]_{cr\atop m=m_i}$ can be found from the following:

$$\left[\left(\bar{N}_x\right)_c\right]_{cr\atop m=m_i} = \frac{m_i^2\pi^2 D_{11}}{L^2} + \frac{\sqrt{t\bar{t}_x}}{\sqrt{3(1-\nu^2)}}\left(\frac{Et}{R}\right)\left\{\frac{\eta_p\sqrt{\gamma}}{2\alpha\beta^2} + \frac{1}{4\alpha\beta^4} + \frac{\alpha\beta^4\left[H - \frac{F}{\beta^2}\right]^2}{\left[1 + 2\eta_s\beta^2 + \beta^4\right]}\right\}_{\substack{\text{Minimum}\\ \text{for } m=m_i}} \tag{2-19}$$

For any given m_i value, the quantity $\left[\left(\bar{N}_x\right)_c\right]_{cr\atop m=m_i}$ is found by minimizing the bracketed expression with respect to the waveform parameter β. Then the particular m_i value of final interest is that which yields the lowermost value for $\left[\left(\bar{N}_x\right)_c\right]_{cr\atop m=m_i}$. This is, in fact, the critical buckling load for the structure and may be denoted by the symbol $\left[\left(\bar{N}_x\right)_c\right]_{cr}$. The corresponding stress value could then be identified simply as σ_{cr}.

In order to express equation (2-19) in terms of stress, one may divide through by $\bar{t}_x$ to obtain

$$\sigma_{cr_{L'=L'_i}} = \frac{\pi^2 E}{\left(\frac{L'_i}{\rho_x}\right)^2} + \frac{E}{\sqrt{3(1-\nu^2)}} \frac{1}{\left(\frac{R}{t}\right)} N^* \tag{2-20}$$

where

L' = Effective length $\left(= \frac{L}{m}\right)$

$$L'_i = \frac{L}{m_i}$$

$$\rho_x = \sqrt{\frac{\bar{I}_x}{\bar{t}_x}} \tag{2-21}$$

and

$$N^* = \frac{1}{\left(\frac{\bar{t}_x}{t}\right)^{1/2}} \left\{ \frac{\eta_p \sqrt{\gamma}}{2\alpha\beta^2} + \frac{1}{4\alpha\beta^4} + \frac{\alpha\beta^4 \left[H - \frac{F}{\beta^2}\right]^2}{\left[1 + 2\eta_s\beta^2 + \beta^4\right]} \right\}_{\substack{\text{Minimum} \\ \text{for } L'=L'_i}} \tag{2-22}$$

From the arrangement of equation (2-20), it is useful to think of the total compressive strength of the cylinder as the sum of two separate components. With this in mind, observe that the first term in equation (2-20) is of the same form as the familiar Euler equation for columns. However, it must also be observed that, unlike the case for columns, this term need not be restricted to the condition that $m_i^2 \leq 4$. For the cylinder, the particular m_i^2 value of interest is that which minimizes equation (2-20) in its entirety and, in the case of long cylinders, shell-type influences can result in buckle patterns with m_i^2 considerably in excess of four. The difference between these two situations is an outgrowth of the fact that, for the column, the critical m_i^2 value is dependent solely upon the end conditions. On the other hand, equation (2-20) was developed for the particular case of a cylindrical shell having simply supported boundaries. Hence, the critical

m_i^2 value of equation (2-20) is a function only of the internal shell stiffnesses. However, the methods of this volume make use of the m_i^2 influence to provide an approximate engineering approach to the analysis of longitudinally stiffened cylinders having various edge conditions. A rigorous solution for cases other than simple support is beyond the scope of the investigation covered here. In particular, the nature of the end conditions is expressed in the form of a fixity factor C_F . This value is taken to be the same as that which the existing boundaries would furnish to ordinary columns. Then the search for critical conditions begins with $m_i^2 = C_F$ and only considers cases where $m_i^2 \geq C_F$. In reality, most of the longitudinally stiffened circular cylinders of practical interest will fall into the relatively short category for which the critical loading corresponds to $m_i^2 = C_F$.

The column-type component of equation (2-20) is usually referred to as a wide-column contribution since the broad circumferential extent of the shell wall precludes buckling about radial axes. That is, the primary buckling displacements are in the radial directions. Tangential buckling displacements are usually of secondary importance, particularly when $n \geq 2$. In order to recognize the influence of the crippling stress for the local wall cross section, the Johnson parabola concept was applied to the wide-column component under discussion. The following expression results:

$$\left(\sigma_{cr}\right)_{L'=L_i'} = \sigma_{cc} - \frac{\sigma_{cc}^{2}\left(L_i'/\rho_x\right)^2}{4\pi^2 E} + \frac{E}{\sqrt{3(1-\nu^2)}}\frac{1}{\left(\frac{R}{t}\right)} N^* \qquad (2\text{-}23)$$

Then to facilitate the application of equations (2-20) and (2-23) in the nonlinear range of the stress-strain curve, the tangent modulus was introduced as follows:

$$\sigma_{cr\,L'=L_i'} = \frac{\pi^2 E_{tan}}{\left(\frac{L_i'}{\rho_x}\right)^2} + \frac{E_{tan}}{\sqrt{3(1-\nu^2)}}\frac{1}{\left(\frac{R}{t}\right)} N^* \qquad (2\text{-}24)$$

and

$$\sigma_{cr_{L'=L'_i}} = \sigma_{cc} - \frac{\sigma_{cc}^2 \left(L'_i/\rho_x\right)^2}{4\pi^2 E} + \frac{E_{tan}}{\sqrt{3(1-\nu^2)}} \frac{1}{\left(\frac{R}{t}\right)} N^* \qquad (2\text{-}25)$$

where

E_{tan} = Tangent modulus

E = Young's modulus

Equation (2-25) applies only where both of the following conditions are satisfied:

$$\text{(a)} \quad \left(\frac{L'_i}{\rho_x}\right) < (\sqrt{2})\,(\pi)\left(\sqrt{\frac{E}{\sigma_{cc}}}\right) \qquad (2\text{-}26)$$

$$\text{(b)} \quad \left\{\begin{matrix}\text{Results from} \\ \text{Equation (2-25)}\end{matrix}\right\} < \left\{\begin{matrix}\text{Results from} \\ \text{Equation (2-24)}\end{matrix}\right\} \qquad (2\text{-}27)$$

For all other situations, equation (2-24) is the applicable formulation. For the linear portion of the stress-strain curve, condition (a) is a sufficient test for the applicability of equation (2-25).

Equations (2-22), (2-24), and (2-25) are the expressions which were used to develop the buckling curves of SECTION 5 and APPENDIX A. In addition, the digital computer programs of SECTION 7 employ these same relationships. The programmed operations for computing N* are fully documented in reference 8. In general, these computations involve the minimization denoted by equation (2-22). That is, for fixed values of L', the indicated function was minimized with respect to β. Both axisymmetric and checkerboard buckling modes were considered. In order to present this output in the most useful form, it was helpful to substitute equations (2-4) into equations (2-3), (2-6), and (2-7) to arrive at the following formulations for the parameters $(\eta_p\sqrt{\gamma})$, α, H, F, and η_s :

$$\left(\eta_p\sqrt{\gamma}\right) = \frac{1}{\left(\frac{\bar{t}_x}{t}\right)^{1/2}}$$

$$\alpha = \frac{\sqrt{3(1-\nu^2)}}{\pi^2}\left(\frac{L'}{R}\right)^2\left(\frac{R}{t}\right)\frac{1}{\left(\frac{\bar{t}_x}{t}\right)^{1/2}}$$

$$H = 1 + \frac{(\nu\pi^2)}{\left(\frac{L'}{R}\right)^2}\left(\frac{\bar{z}_x}{R}\right) \tag{2-28}$$

$$F = \frac{\pi^2}{\left(\frac{L'}{R}\right)^2}\left(\frac{\bar{t}_x}{t}\right)^{1/2}\left(\frac{\bar{z}_x}{R}\right)$$

$$\eta_s = \frac{-\nu}{\left(\frac{\bar{t}_x}{t}\right)^{1/2}} + (1+\nu)\left(\frac{\bar{t}_x}{t}\right)^{1/2}$$

Reference 8 shows all of the detailed algebra involved in the derivation of these formulas.

As noted above, equations (2-24) and (2-25) incorporate the tangent modulus E_{tan} to account for nonlinearity in the applicable stress-strain curve. In order to establish appropriate values for this modulus, digital computer program 4196 (see SECTION 7) and the critical stress curves of SECTION 5.1 and APPENDIX A make use of the Ramberg-Osgood [9] representation of the stress-strain curve. For the particular cases included in this volume, the following values were used for the Ramberg-Osgood parameters:

Material	Ramberg-Osgood n	Ramberg-Osgood $\sigma_{.7}$
7075-T6 Aluminum Alloy	10	70,000 psi
6Al-4V Titanium Alloy	35	133,500 psi
718 Nickel Alloy	12.7	150,500 psi

Digital computer program 4196 can accommodate different materials by simple changes in these input values and the input Young's modulus.

In conclusion of this section, attention is directed to the fact that the lengths L and L' appear in many of the equations presented here. When expressed in these terms the equations apply directly to cylinders which do not incorporate any intermediate rings. The symbol L denotes the overall length of such cylinders. However, in the absence of general instability (see GLOSSARY, Volume I [1]),these same equations can be applied to the sections which lie between rings in cylinders having both axial and circumferential stiffening. To accomplish this it is only necessary to

(a) Replace L with the ring spacing a

and/or

(b) Replace L' with $a'(=\frac{a}{m})$ where m is now the number of axial half-waves between two rings.

SECTION 3

TEST DATA COMPARISONS

3.1 GENERAL

The methods proposed in this volume for the analysis of longitudinally stiffened cylinders were evaluated by comparing computed critical stress values against the test data of references 10 through 14. The results from these investigations are given in Tables I through V. The computed critical stresses were obtained by using the digital computer programs of SECTION 7. These are the programs that were used to generate the buckling curves presented in SECTIONS 5.1, 5.2, and APPENDIX A. Except for the specimens of reference 12 all of the data reported here were obtained from cylinders which incorporated stiffener eccentricities (see GLOSSARY, Volume I [1]). This influence was fully accounted for in the tabulated predictions.

For the purposes of test data comparisons, the most appropriate knock-down criteria are the 50% probability curves given in Volume V [2]. For all of the specimens listed in Tables I through III, the related knock-down factor was found to be essentially equal to unity. The tabulated results for these specimens are based upon this value. From the comparison ratios shown in Tables I and II, it can be seen that, relative to the data of references 10 and 11, the proposed analytical methods gave conservative predictions. Since these ratios are not clouded by knock-down factors less than unity, they would seem to reveal an apparently inherent conservatism in the fundamental theory applied here. This conservatism is at least partly due to the fact that the analysis neglects the torsional stiffnesses of the stringers. In addition, for the most part, the proposed approach treats the wavelength parameter β as a continuous variable. Limits are set which disallow buckle patterns for which $0 < n < 1$. However, aside from this, restrictions to integral numbers of circumferential full-waves are not enforced. This introduces some conservatism through a neglect of cusp-like patterns in the buckling curves. Another possible source of conservatism lies in the engineering approximation used to account for boundary conditions other than simple support. However, further study would be required to establish that this is actually the case.

Although the test data of references 10 and 11 revealed a conservative trend in the prediction techniques, it is noted from Table III that the data of Cheatham [12] display scatter on either side of the related predictions. This is due to the fact that the inherent conservatism cited above is embodied in the shell contribution to the total compressive strength. Since their corrugated walls have very little extensional stiffness, the specimens of reference 12 receive virtually no contribution from shell behavior. This is reflected into the analysis through the N^* values which become very small. The prediction equations then reduce essentially to the familiar Euler-Johnson relationships. Hence, in these cases, deviations from predicted strengths are due largely to individual geometric variations among the test specimens, maldistribution of load, etc.

In the case of the specimens from reference 13, the test results are quite close to the predictions if the applicable knock-down factor is assumed to be unity. However, for these specimens the (R/t_{eff}) ratio is sufficiently high for the 50% probability curves of Volume V [2] to give much smaller Γ values. The comparison ratios (Calculated σ_{cr} ÷ Test σ_{cr}) then reduce to .72 and .90 for the two tests considered here. The related test reports indicate that extreme care was taken to minimize imperfections in these specimens so that the applied knock-down factor (.64) is probably unduly severe for these particular tests.

In the case of the specimens from reference 14, once again the (R/t_{eff}) ratios are sufficiently high for the 50% probability curves of Volume V [2] to give Γ values considerably less than unity. In these tests the built-in specimen imperfections were probably more typical of those likely to be encountered in practical applications. As shown in Table V, the comparison ratios (Calculated σ_{cr} ÷ Test σ_{cr}) display a reasonable degree of scatter on either side of 1.0. The lowest value is 0.93 and the highest is 1.19. For actual application of the proposed methods, one would, of course, use 90% or 99% probability knock-down factors to obtain safe design levels.

In each of the Tables I through V, the quantity L is the overall length of the cylinder between end supports. In addition, all of the tabulated σ_{cr} values were obtained by dividing the total critical axial load by the total cross-sectional area of the specimens. Except for the specimens of reference 13, the fixity factor C_F ($= m_i^2$) was taken equal to 3.75. This value is cited by Peterson and Dow [10] as a common choice for the apparent fixity coefficient for flat-end column tests. For the specimens of reference 13, the fixity factor C_F was taken equal to 2.5. This value was experimentally determined by concurrent control tests on wide columns having the same wall cross section and boundary constraint as the test cylinders.

3.2 TESTS OF PETERSON AND DOW [10]

The comparisons of predictions versus test results for these specimens are given in Table I. All of these tests were performed on cylinders having Z - shaped longitudinal stiffeners which were riveted to the basic cylindrical skin. No buckling of the isotropic skin panels and no local buckling of the longitudinal stiffeners occurred prior to the overall instability. All of these specimens were made of 7075-T6 aluminum alloy for which the following properties were assumed to apply:

	E	$= 10.5 \times 10^6$ psi
	ν	$= .33$
Ramberg-Osgood	n	$= 10$
Ramberg-Osgood	$\sigma_{.7}$	$= 70{,}000$ psi
	σ_{cy}	$= 67{,}000$ psi

TABLE I - Comparison of Calculations vs. Test Data of Ref. 10

Specimen	Stringer Location	R in.	L in.	$\left(C_F = m_i^2\right)$	Γ	σ_{cc} 10^3 psi	N*	$\left(\frac{R}{t}\right)$	$\left(\frac{L'}{\rho_x}\right)$	Calculated σ_{cr} 10^3 psi	Test σ_{cr} 10^3 psi	$\left(\frac{\text{Calculated } \sigma_{cr}}{\text{Test } \sigma_{cr}}\right)$
1	Internal	23.86	25.40	3.75	1	59.2	.1964	604	70.2	23.0	25.0	.92
2	Internal	23.86	19.28	3.75	1	59.2	.1558	604	53.3	36.7	38.2	.96
3	Internal	15.92	29.68	3.75	1	59.2	.275	395	82.0	19.9	23.8	.84
4	Internal	15.92	20.77	3.75	1	59.2	.201	397	57.4	34.1	38.0	.90
5	Internal	15.92	41.83	3.75	1	59.2	.354	397	115.6	13.5	17.2	.78
6	Internal	15.92	59.00	3.75	1	59.2	.433	397	163.1	10.9	13.7	.80

The crippling stress was computed by considering all elements of the shell wall cross section to be flat plates. The crippling stress for each individual element was determined from Figure C 1.3.1-13 of reference 15. The crippling stress for the total wall was taken as the weighted average of these individual values. The section properties $\bar{t}_x$, $\bar{I}_x$, and ρ_x of the shell wall (stringers plus skin) were assumed to be the same for all of the specimens of reference 10. In particular, these values were computed to be

$$\bar{t}_x = .0737 \frac{in^2}{in}$$

$$\bar{I}_x = .00258 \frac{in^4}{in}$$

$$\rho_x = \sqrt{\frac{.00258}{.0737}} = .1868 \text{ in}$$

They are based on the assumption that all of the skin and stringer material was fully effective.

3.3 TESTS OF CARD [11]

The comparisons of predictions versus test results for these specimens are given in Table II. Specimens 1 through 4 were all integrally stiffened by solid rectangular stringers while specimen 5 had Z-shaped stringers which were riveted to the basic cylindrical skin. No buckling of the isotropic skin panels and no local buckling of the longitudinal stiffeners occurred prior to the overall instability. Specimens 1 through 4 were made of 2024-T351 aluminum alloy for which the following properties were assumed to apply:

TABLE II - Comparison of Calculations vs. Test Data of Ref. 11

Specimen	Stringer Location	R in.	L in.	$C_F \left(= \frac{m_i^2}{2} \right)$	Γ	σ_{cc} 10^3 psi	N*	$\left(\frac{R}{t}\right)$	$\left(\frac{L'}{\rho_x}\right)$	Calculated σ_{cr} 10^3 psi	Test σ_{cr} 10^3 psi	$\left(\frac{\text{Calculated } \sigma_{cr}}{\text{Test } \sigma_{cr}}\right)$
1	External	9.55	38.00	3.75	1	47.5	1.071	338	189.8	21.6	30.5	.71
2	Internal	9.55	38.00	3.75	1	47.5	.387	345	189.8	10.1	12.9	.78
3	External	9.55	23.75	3.75	1	47.5	1.306	347	118.6	24.7	34.4	.72
4	Internal	9.55	23.75	3.75	1	47.5	.286	341	118.6	12.8	17.0	.75
5	External	15.80	59.00	3.75	1	59.2	1.107	385	163.2	22.2	23.7	.94

	E	$= 10.5 \times 10^6$ psi
	ν	$= .33$
Ramberg-Osgood	n	$= 10$
Ramberg-Osgood	$\sigma_{.7}$	$= 37{,}000$ psi
	σ_{cy}	$= 38{,}000$ psi

Specimen 5 was made of 7075-T6 aluminum alloy for which the properties cited in SECTION 3.2 above were assumed to apply. For all of the specimens the crippling stresses were computed by considering the elements of the shell wall cross section to be flat plates. The crippling stress for each individual element was determined from Figure C 1.3.1-13 of reference 15 The crippling stress for an entire shell wall was taken as the weighted average of the appropriate individual values. The section properties $\bar{t}_x$, $\bar{I}_x$, and ρ_x of the shell wall (stringers plus skin) were assumed to be the same for specimens 1 through 4. These were computed to be

$$\bar{t}_x = .0573 \frac{in^2}{in}$$

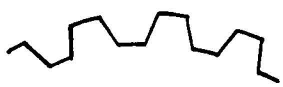
$$\bar{I}_x = .000612 \frac{in^4}{in}$$

$$\rho_x = \sqrt{\frac{.000612}{.0573}} = .1033 \text{ in}$$

These values are based on the assumption that all of the skin and stringer material was fully effective. For specimen 5, the section properties $\bar{t}_x$, $\bar{I}_x$, and ρ_x of the shell wall were taken equal to the respective values cited in SECTION 3.2 above. As noted there, these values likewise assume that all of the skin and stringer material was fully effective.

3.4 TESTS OF CHEATHAM [12]

The comparisons of predictions versus test results for these specimens are given in Table III. In order that like geometries might be grouped together, the specimens are not listed in numerical sequence. This manner of presentation permits one to observe the degree of scatter that exists among the test data obtained from nominally identical specimens. Each of these specimens consisted of corrugated walls which were joined to heavy end rings by casting Woods' Metal into annular grooves that held the corrugations in place. No intermediate rings were used. Figure 3 shows the two types of local wall cross sections that were tested. For specimens



(Specimens 1-33)

(Specimens 34-45)

Figure 3 - Corrugation Configurations of Reference 12

1 through 33, the cross section was essentially composed of intersecting flats while, for specimens 34 through 45, the corrugation was essentially of a sine-wave form. Obviously, shell walls of these types do not incorporate any eccentricities (see GLOSSARY, Volume I [1]). All of these specimens were made of 5086-H34 aluminum alloy for which the following properties were assumed to apply:

$E = 10.8\times10^6$ psi (From test data given in ref. 12)

$\nu = .35$

Ramberg-Osgood $n = 43.8$ (Calculated from stress-strain curve given in ref. 12)

Ramberg-Osgood $\sigma_{.7} = 34{,}200$ psi (Obtained from stress-strain curve given in ref. 12)

$\sigma_{cy} = 34{,}400$ psi (From test data given in ref. 12)

TABLE III - Comparison of Calculations vs. Test Data of Ref. 12

Specimen	R in.	L in.	$C_F \left(= m_i^2 \right)$	Γ	σ_{cc} 10^3 psi	N*	$\left(\frac{R}{t}\right)$ (See discussion in SECTION 3.4)	$\left(\frac{L'}{\rho_x}\right)$	Calculated σ_{cr} 10^3 psi	Test σ_{cr} 10^3 psi	$\left(\frac{\text{Calculated } \sigma_{cr}}{\text{Test } \sigma_{cr}}\right)$
1	10.38	3	3.75	1	34.0	.00228	1,445	21.7	32.6	27.8	1.17
4	10.38	3	3.75	1	34.0	.00228	1,445	21.7	32.6	32.0	1.02
12	10.38	3	3.75	1	34.0	.00228	1,445	21.7	32.6	34.5	.94
21	10.38	3	3.75	1	34.0	.00228	1,445	21.7	32.6	33.0	.99
2	10.38	5	3.75	1	34.0	.00633	1,445	36.2	30.4	29.5	1.03
19	10.38	5	3.75	1	34.0	.00633	1,445	36.2	30.4	31.0	.98
3	10.38	11	3.75	1	34.0	.0306	1,445	79.6	17.0	15.5	1.10
5	10.38	11	3.75	1	34.0	.0306	1,445	79.6	17.0	14.2	1.20
6	10.38	9	3.75	1	34.0	.0205	1,445	65.1	22.5	18.8	1.20
20	10.38	9	3.75	1	34.0	.0205	1,445	65.1	22.5	19.5	1.15
7	10.38	4	3.75	1	34.0	.00406	1,445	28.9	31.7	34.3	.92
13	10.38	4	3.75	1	34.0	.00406	1,445	28.9	31.7	33.0	.96
32	10.38	4	3.75	1	34.0	.00406	1,445	28.9	31.7	32.8	.97
8	10.38	7	3.75	1	34.0	.01242	1,445	50.7	27.0	23.3	1.16
15	10.38	7	3.75	1	34.0	.01242	1,445	50.7	27.0	25.5	1.06
33	10.38	7	3.75	1	34.0	.01242	1,445	50.7	27.0	25.1	1.08
9	10.38	8	3.75	1	34.0	.01621	1,445	57.9	24.9	20.9	1.19
17	10.38	8	3.75	1	34.0	.01621	1,445	57.9	24.9	23.4	1.06
18	10.38	8	3.75	1	34.0	.01621	1,445	57.9	24.9	23.6	1.06
10	10.38	6	3.75	1	34.0	.00912	1,445	43.4	28.8	26.0	1.11
16	10.38	6	3.75	1	34.0	.00912	1,445	43.4	28.8	28.5	1.01
11	10.38	2	3.75	1	34.0	.001014	1,445	14.46	33.2	31.9	1.04
14	10.38	15	3.75	1	34.0	.0570	1,445	108.5	9.32	8.57	1.09

TABLE III - Comparison of Calculations vs. Test Data of Ref. 12 (Cont'd)

Specimen	R in.	L in.	$C_F \left(= \frac{m_i}{2} \right)$	Γ	σ_{cc} 10^3 psi	N*	$\left(\frac{R}{t}\right)$ (See discussion in SECTION 3.4)	$\left(\frac{L'}{\rho_x}\right)$	Calculated σ_{cr} 10^3 psi	Test σ_{cr} 10^3 psi	$\left(\frac{\text{Calculated } \sigma_{cr}}{\text{Test } \sigma_{cr}}\right)$
22	6.00	4	3.75	1	34.0	.00701	836	28.9	31.7	33.4	.95
23	7.50	4	3.75	1	34.0	.00561	1,045	28.9	31.7	34.0	.93
24	9.00	4	3.75	1	34.0	.00468	1,253	28.9	31.7	32.0	.99
25	6.00	7	3.75	1	34.0	.0215	836	50.7	27.1	28.2	.96
26	7.50	7	3.75	1	34.0	.01719	1,045	50.7	27.0	27.8	.97
27	9.00	7	3.75	1	34.0	.01431	1,253	50.7	27.0	26.7	1.01
28	6.00	9.5	3.75	1	34.0	.0395	836	68.8	21.5	20.1	1.07
29	9.00	10	3.75	1	34.0	.0292	1,253	72.4	19.9	17.0	1.17
30	7.50	9.5	3.75	1	34.0	.0317	1,045	68.8	21.4	21.3	1.00
31	10.38	10	3.75	1	34.0	.0253	1,445	72.4	19.9	15.7	1.27
34	10.38	2	3.75	1	28.0	.001992	1,400	17.36	27.4	27.6	.99
35	10.38	2	3.75	1	28.0	.001992	1,400	17.36	27.4	26.3	1.04
36	10.38	3	3.75	1	28.0	.00447	1,400	26.1	26.7	26.8	1.00
37	10.38	3.5	3.75	1	28.0	.00609	1,400	30.4	26.2	27.6	.95
38	10.38	4	3.75	1	28.0	.00796	1,400	34.8	25.7	27.3	.94
39	10.38	5.5	3.75	1	28.0	.01504	1,400	47.8	23.8	26.7	.89
40	10.38	7	3.75	1	28.0	.0244	1,400	60.8	21.3	22.9	.93
41	10.38	8	3.75	1	28.0	.0318	1,400	69.5	19.3	17.9	1.08
42	10.38	9	3.75	1	28.0	.0403	1,400	78.2	16.9	16.1	1.05
43	10.38	10.5	3.75	1	28.0	.0548	1,400	91.2	13.1	12.6	1.04
44	10.38	12.5	3.75	1	28.0	.0777	1,400	108.5	9.42	9.90	.95
45	10.38	15	3.75	1	28.0	.1119	1,400	130.2	6.82	8.67	.79

The crippling stress values were selected on the basis of the test data reported in reference 12 for specimens having lengths of 3 inches or less. As shown in that reference, these specimens failed in the crippling mode. The selected values were then substantiated by approximate calculations. For this purpose, both types of corrugations were assumed to be composed solely of flat elements. The sine-wave configuration was approximated by a saw-tooth pattern. Crippling stresses for individual flat elements were determined from Figure C 1.3.1-13 of reference 15. The crippling stress for an entire corrugation was taken as the weighted average of these individual values. The calculated values showed good agreement with those selected from the test data. The latter were used in the analysis. The section properties $\bar{t}_x$, $\bar{I}_x$, and ρ_x of the corrugated walls were computed to be as follows:

<u>Specimens 1-33</u>	<u>Specimens 34-45</u>
$\bar{t}_x = .01262 \frac{in^2}{in}$	$\bar{t}_x = .01168 \frac{in^2}{in}$
$\bar{I}_x = .0000642 \frac{in^4}{in}$	$\bar{I}_x = .0000412 \frac{in^4}{in}$
$\rho_x = \sqrt{\frac{.0000642}{.01262}} = .0713 \text{ in.}$	$\rho_x = \sqrt{\frac{.0000412}{.01168}} = .0594 \text{ in.}$

These values are based on the assumption that all of the corrugation material was fully effective.

In SECTION 4 it is recommended that, for the analysis of corrugated cylinders, one may obtain realistic critical stress values by setting N^* equal to zero. This recommendation is based upon the promise that the accordion-like hoop flexibility of the corrugations virtually eliminates shell-type contributions to the total strength. Nevertheless, for the analysis of the reference 12 test specimens, actual N^* values were computed through the substitution of elastic constants into digital computer program 4235. SECTION 4 includes some discussion of the procedures required in this regard. The resulting finite N^* values were used in computing the predicted stresses listed in Table III. It was noted that these N^* values are indeed very small and that the associated contributions to the overall strengths are negligible for all the specimens considered. Thus, substantiation was obtained for the recommended practice of setting $N^* = 0$ for corrugated walls.

3.5 TESTS OF LOCKHEED [13]

The comparisons of predictions versus test results for these specimens are given in Table IV. Both of these tests were performed on circular panels which were integrally stiffened by solid rectangular stringers. The panels were curved to a radius of 198 inches and had a total circumferential arc length of 109 inches. It was experimentally established that these panels were sufficiently wide for them to have behaved essentially as full circular cylinders. No buckling of the isotropic skin panels and no local buckling of the longitudinal stiffeners occurred prior to the overall instability. Both specimens were made of 2219-T87 aluminum alloy for which the following properties were assumed to apply:

$$E = 10.8 \times 10^6 \text{ psi}$$
$$\nu = .35$$
$$\text{Ramberg-Osgood } n = 10$$
$$\text{Ramberg-Osgood } \sigma_{.7} = 49{,}700 \text{ psi}$$
$$\sigma_{cy} = 53{,}000 \text{ psi}$$

The crippling stress was computed by considering all elements of the shell wall cross section to be flat plates. The crippling stress for each individual element was determined from Figure C 1.3.1-13 of reference 15 The crippling stress for the total wall was taken as the weighted average of these individual values. The section properties $\bar{t}_x$, $\bar{I}_x$, and ρ_x of the shell wall (stringers plus skin) were assumed to be the same for both specimens. These were computed to be

$$\bar{t}_x = .223 \frac{in^2}{in}$$

$$\bar{I}_x = .0520 \frac{in^4}{in}$$

$$\rho_x = \sqrt{\frac{.0520}{.223}} = .483 \text{ in}$$

These values are based on the assumption that all of the skin and stringer material was fully effective.

3.6 TESTS OF KATZ [14]

The comparisons of predictions versus test results for these specimens are given in Table V. All of these tests were performed on cylinders which had rather shallow integral stiffening. Specimens 5A-1, 5A-2, 5B-1, and 5B-2 had solid rectangular stiffeners while, for specimens 5C-1 and 5C-2, the stiffeners were T-shaped. No buckling of the isotropic skin panels and no local buckling of the longitudinal stiffeners occurred prior to the overall instability. All of the specimens were made of 6061-T6 aluminum alloy for which the following properties were assumed to apply:

	E	$= 10.5 \times 10^6$ psi
	ν	$= .325$
Ramberg-Osgood	n	$= 31$
Ramberg-Osgood	$\sigma_{.7}$	$= 35{,}000$ psi
	σ_{cy}	$= 35{,}000$ psi

The crippling stresses were computed by considering the elements of the shell wall cross section to be flat plates. The crippling stress for each individual element was determined from Figure C 1.3.1-13 of reference 15. The crippling stress for an entire shell wall was taken as the weighted average of the appropriate individual values. The section properties $\bar{t}_x$, $\bar{I}_x$, and ρ_x were computed to be as follows:

TABLE IV - Comparison of Calculations vs. Test Data of Ref. 13

Specimen	Stringer Location	R in.	L in.	$C_F \left(= \frac{m_1^2}{}\right)$	Γ	σ_{cc} 10^3 psi	N*	$\left(\frac{R}{t}\right)$	$\left(\frac{L'}{\rho_x}\right)$	Calculated σ_{cr} 10^3 psi $\Gamma = 1$	Calculated σ_{cr} 10^3 psi $\Gamma = .64$	Test σ_{cr} 10^3 psi	$\left(\frac{\text{Calculated } \sigma_{cr}}{\text{Test } \sigma_{cr}}\right)$ (Γ = .64)
I-A-3	External	198	95	2.5	.64	37.8	1.503	1,215	124	15.1	12.2	16.9	.72
III-A-1	Internal	198	95	2.5	.64	37.8	.394	1,215	124	9.0	8.3	9.2	.90

TABLE V - Comparison of Calculations vs. Test Data of Ref. 14

1	2	3	4	5	6	7	8	9	10	11		12	13
Specimen	Stringer Location	R in.	L in.	$C_F \left(= \frac{m_1^2}{}\right)$	Γ	σ_{cc} 10^3 psi	N*	$\left(\frac{R}{t}\right)$	$\left(\frac{L'}{\rho_x}\right)$	Calculated σ_{cr} 10^3 psi $\Gamma = 1$	Calculated σ_{cr} 10^3 psi Γ from column 6	Test σ_{cr} 10^3 psi	$\left(\frac{\text{Calculated } \sigma_{cr}}{\text{Test } \sigma_{cr}}\right)$ (Γ from column 6)
5A-1	External	26.0	57.3	3.75	.54	22.1	.992	694	512	9.56	5.35	5.78	.93
5A-2	External	26.0	57.3	3.75	.54	22.1	.992	694	512	9.56	5.35	5.48	.98
5B-1	External	26.0	57.3	3.75	.58	28.3	.978	685	480	9.60	5.76	5.23	1.10
5B-2	External	26.0	57.3	3.75	.58	28.3	.978	685	480	9.60	5.76	5.03	1.15
5C-1	External	26.0	57.3	3.75	.66	29.6	1.001	676	293	10.69	7.46	6.27	1.19
5C-2	External	26.0	57.3	3.75	.66	29.6	1.001	676	293	10.69	7.46	7.48	.997

Specimens	$\bar{t}_x$ in^2/in	$\bar{I}_x$ in^4/in	ρ_x in
5A-1 and 5A-2	.0468	.0001555	.0577
5B-1 and 5B-2	.0511	.0001932	.0615
5C-1 and 5C-2	.0573	.000583	.1009

These values are based on the assumption that all of the skin and stringer material was fully effective.

SECTION 4

ANALYSIS METHOD

As noted earlier, the methods of this volume apply equally well to cylinders which incorporate only longitudinal stiffening and to sections which lie between rings in cylinders incorporating both axial and hoop stiffening (stringers and rings). Application to the latter case is only valid where general instability (see GLOSSARY, Volume I [1]) does not precede the panel instability mode (see GLOSSARY, Volume I [1]).

The given methods employ the smearing-out technique whereby discrete stiffness values are averaged over the entire surface of the cylinder. One must therefore exercise engineering judgement to prevent misapplication to configurations having excessively large stringer spacings. When wide spacings are encountered, one might choose to refine the methods of this report through the introduction of effectivity concepts.

In essence, the primary analysis method of this volume consists of the following two basic steps:

(a) By using the curves given in Figure 7, establish an appropriate value for the minimization factor N^*.

(b) By using the above N^* value in conjunction with the curves given in Figures 4, 5, and 6, find the buckling stress.

In order to avoid the need for interpolation, one might choose to use digital computer programs 4196 and 4235 (see SECTION 7) instead of the curves cited above. In addition, note that APPENDIX A presents the buckling data of Figures 4, 5, and 6 in a slightly different format which might better suit the personal preferences of the user.

It is important to note here that a Donnell-type theory furnishes the basis for the methods given in this volume. Therefore, these methods cannot be applied to cases of non-axisymmetric buckling where the number of circumferential full-waves is small. As a rule-of-thumb, it is suggested that the methods be considered inapplicable where

$$0 < n < 2 \qquad (4\text{-}1)$$

All of the curves given in this volume conpletely ignore this restriction. Therefore, to insure that this condition is not violated, one should supplement the plotted data with appropriate checks from digital computer runs, using the programs of SECTION 7. However, most practical configurations likely to be encountered will display buckle patterns having $n \geq 2$. Hence it should not prove necessary to make the suggested check for every configuration investigated. When a large number of candidate designs are to be studied, it will usually be reasonable for one to assume that the wave-number restrictions are satisfied so that the foregoing check need only be made as a final operation for a few selected cases.

In addition, it should be noted that the curves and computer programs of this volume are directly applicable only to the relatively short cylinders for which the critical loading condition corresponds to $m^2 = C_F$. Separate checks should be made to establish that this condition is satisfied. Here again, in usual practical applications the restriction will be satisfied so that such checks can normally be made as a final operation for selected cases. To accomplish the check for shortness, one must investigate those situations for which

$$\sigma_{cc} = \infty \text{ and } m^2 \geq C_F \tag{4-2}$$

to verify that the case where $m^2 = C_F$ does indeed give a lowermost stress value. The specification that $\sigma_{cc} = \infty$ is made to insure that only equation (2-24) is used here. Equation (2-25) is eliminated from immediate consideration since the crippling mode plays a cut-off type of role which, it is thought, should not be reflected into the minimization process under discussion. In actual practice, the search indicated above need not involve a large number of m values. Normally, one will be able to conclude that the cylinder is "short" by the inspection of the results corresponding to only 3 or 4 m values. The various values for m are injected into the analysis through the L' value which is computed as follows:

$$L' = \frac{L}{m} \tag{4-3}$$

Whenever it is found that the lowermost stress (with $\sigma_{cc} = \infty$) occurs when

$$m^2 = m_L^2 > C_F \tag{4-4}$$

the desired critical buckling stress can be found from the curves or computer programs by using

$$\begin{aligned} \sigma_{cc} &= \text{Actual for the cross section} \\ &\text{and} \\ L' &= \frac{L}{m_L} \end{aligned} \tag{4-5}$$

The methods of this volume are primarily intended for application to structures which, prior to overall instability, do not experience buckling of the isotropic skin panels and/or local buckling of the longitudinal stiffeners (stringers). However, for any practical cases which actually involve prior buckling of the isotropic skin panels, it is recommended that one proceed by simply taking $N^* = 0$. That is, no shell-type contribution would be considered and the strength equations would simply reduce to the familiar Euler-Johnson relationships. The buckling curves presented in the APPENDIX include the $N^* = 0$ case. On the other hand, when there is no buckling of the isotropic skin panels but local buckling of the stringers occurs, one may proceed by introducing effective-width concepts to arrive at appropriate section properties for the composite shell wall.

The methods of this volume are chiefly directed toward application to conventional skin-stringer constructions. However, one will frequently be interested in cylinders composed solely of corrugated walls. The curves of SECTION 5.2 do not apply to such configurations since these plots are based upon the elastic constant formulas given as equations (2-4). The geometric characteristics peculiar to corrugations require the use of different formulations for these constants (see Table X). Therefore, to compute the critical buckling stress for corrugated walls, one might proceed as follows:

(a) Compute the appropriate elastic constants by using the formulas given in Table X.

(b) Substitute the above values for the elastic constants into digital computer program 4235 to obtain the corresponding output "MINIMUM VALUE".

(c) Compute N* as follows:

$$N^* = \frac{1}{\left(\frac{\bar{t}_x}{t}\right)^{1/2}} \times (\text{"MINIMUM VALUE"}) \tag{4-6}$$

where

$$t = t_c \left(\frac{\delta\theta}{\Delta\theta}\right)^{1/3} = t_c \left(\frac{2\pi R}{\Sigma d_i}\right)^{1/3} \tag{4-7}$$

See Table X for clarification of the terms $\bar{t}_x$, t_c, $\delta\theta$, $\Delta\theta$, and Σd_i. It should be helpful to note that this formulation for t arises out of the fact that it enters into the buckling equation through the elastic constant D_{22}.

(d) Then, by using equation (4-7) to compute the ratio (R/t), enter the curves of SECTION 5.1 or APPENDIX A to arrive at the critical buckling stress.

However, the work entailed in performing these operations will usually prove to be quite unnecessary since the final result will generally show that, for corrugated walls, $N^* \approx 0$. The accordion-like hoop flexibility of the corrugations essentially eliminates shell-type contributions so that the total strength can be computed from the familiar Euler-Johnson relationships. Thus, it is recommended here that, for corrugations, one go directly to the curves in APPENDIX A for the case $N^* = 0$. Since the related stress values are then independent of (R/t), the appropriate value for this ratio is of no importance here. However, in the interest of clarity and correctness, one should use equation (4-7) in this computation. An alternative, equally simple recommendation is that Figures 4, 5, and 6 be used for the analysis of corrugations by taking the buckling stress values from the high $(R/N^{*}t)$ region where the curves essentially become horizontal straight lines.

In conclusion of this section, it is pointed out that this volume essentially gives classical theoretical values which do not account for the influences from initial imperfections. It is therefore recommended that the values obtained from the methods given here be reduced in accordance with the knock-down criterion of Volume V [2]. This should result in design values of high reliability.

SECTION 5

BUCKLING CURVES

5.1 CRITICAL STRESS

The curves of this section present critical stress values for longitudinally stiffened circular cylinders subjected to axial compression. To make proper use of these curves, one should refer to the instructions furnished in SECTION 4, "ANALYSIS METHOD". All of these curves were generated by digital computer program 4196 (see SECTION 7) which was used in conjunction with an automatic plotting machine. Equations (2-24) and (2-25) provide the basis for this program.

The curves of Figures 4, 5, and 6 involve the terms CRIPPLING STRESS (L'/ρ_x), and (R/N*t) where,

CRIPPLING STRESS = σ_{cc} = Crippling stress of the local wall cross section. Conventional methods are readily available for the computation of this value.

L' = An effective length which, for short cylinders, may be computed from the equation

$$L' = \frac{L}{\sqrt{C_F}} \tag{5-1}$$

The quantity C_F is the fixity coefficient which would apply to a wide-column having the same boundary conditions as the actual cylinder. See SECTION 4 concerning certain checks which should be made in connection with the L' value.

ρ_x = The local longitudinal radius of gyration of the shell wall. This quantity may be computed from the equation

$$\rho_x = \sqrt{\frac{\bar{I}_x}{\bar{t}_x}} \tag{5-2}$$

For definitions of $\bar{I}_x$ and $\bar{t}_x$, see the notes following Table X.

R = Radius to middle surface of basic cylindrical skin.

N* = Minimization factor found from the curves of Figure 7 or from digital computer program 4235.

t = Thickness of basic cylindrical skin in conventional skin-stringer constructions. To facilitate application to other configurations, it is helpful to note that this quantity is introduced here through the elastic constant D_{22}. (See the discussion in SECTION 4 concerning application to corrugated cylinders).

5.1.1 7075-T6 ALUMINUM ALLOY

Table VI lists the families provided here for longitudinally stiffened cylinders made of 7075-T6 aluminum alloy. These curves are based upon the following values for the indicated material properties:

$E = 10.5\text{x}10^6$ psi

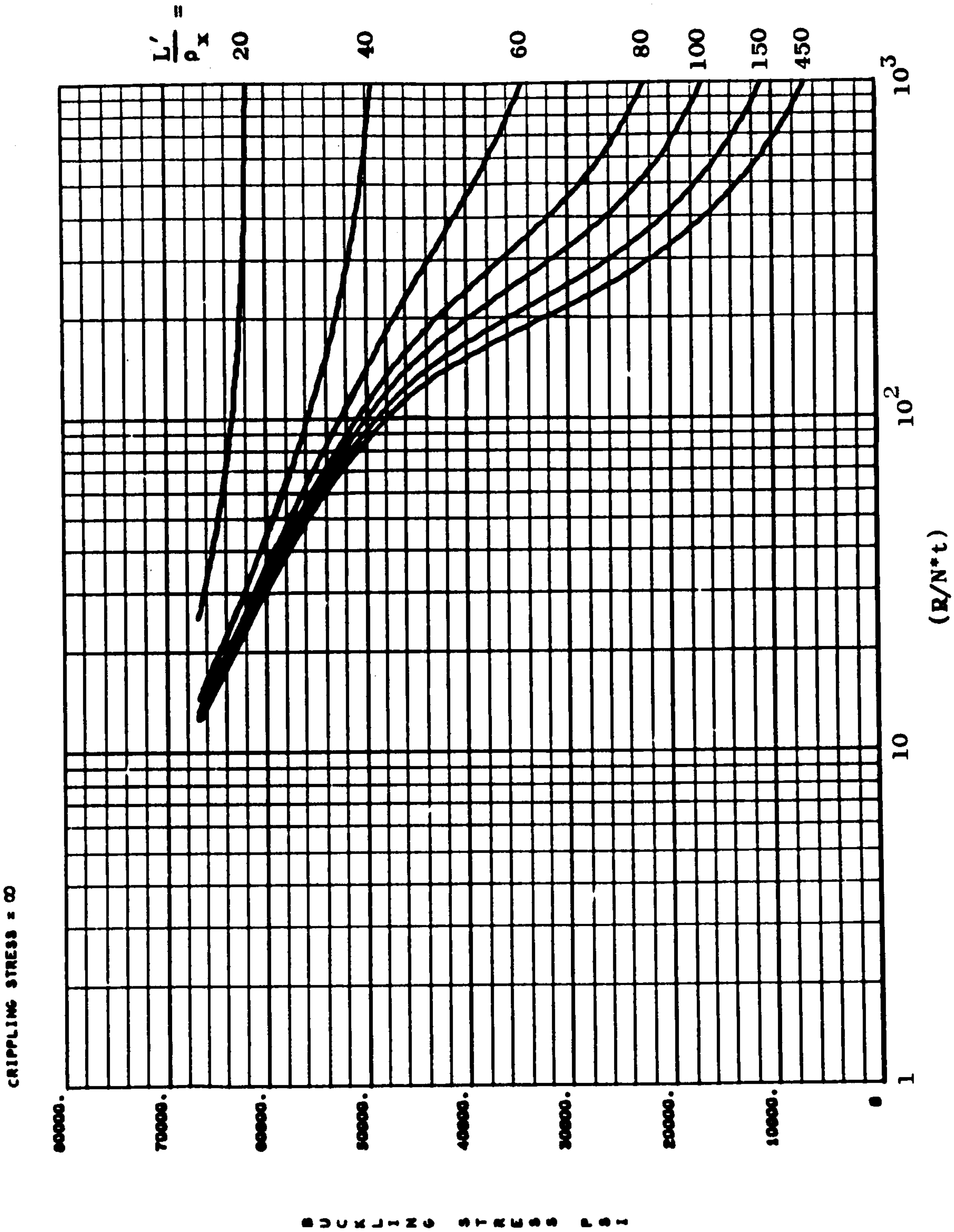
$\nu = .33$

$\sigma_{cy} = 67{,}000$ psi

Ramberg-Osgood $n = 10$

Ramberg-Osgood $\sigma_{.7} = 70{,}000$ psi

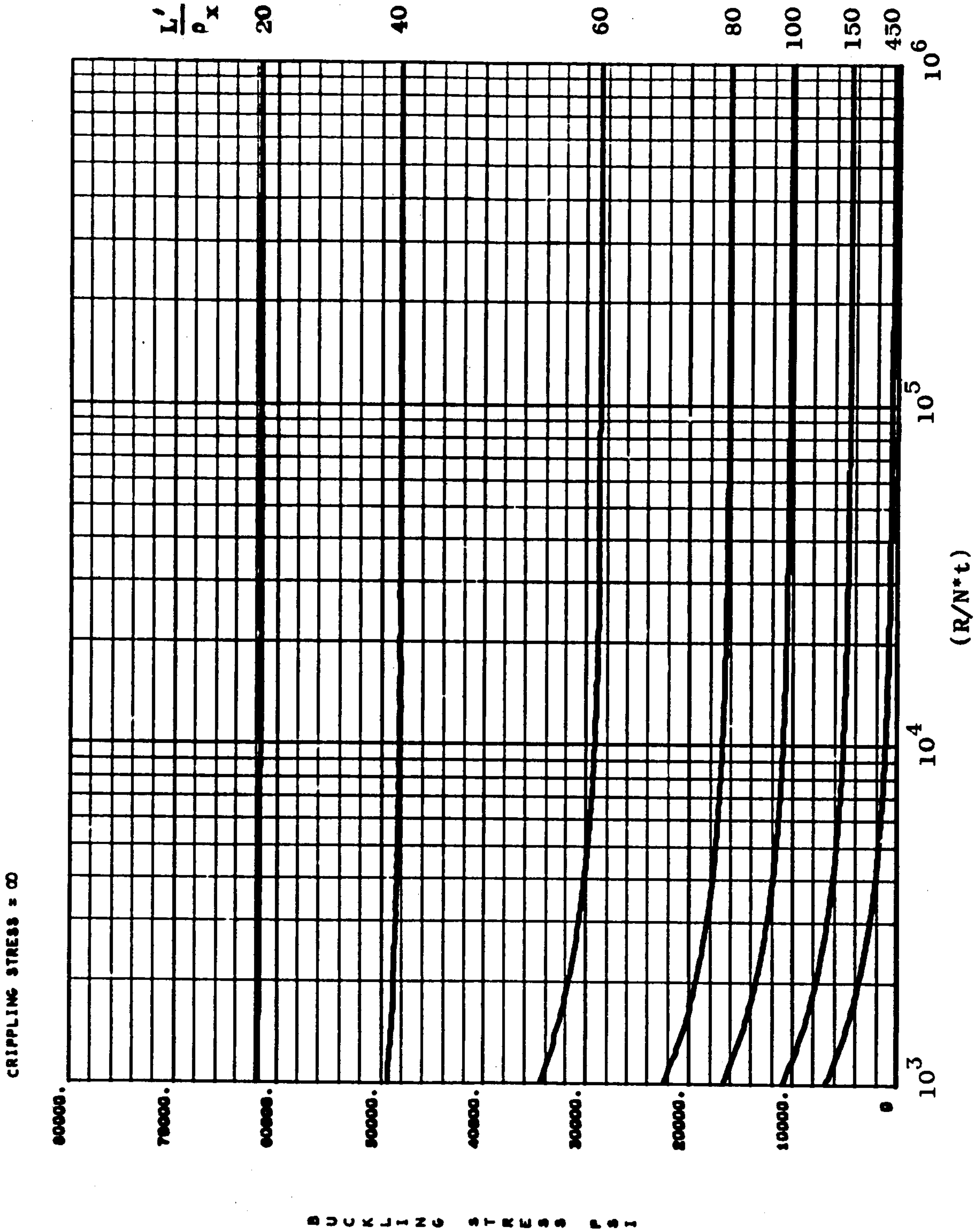
TABLE VI - Table of Contents for Curves of Compressive Buckling Stress for Longitudinally Stiffened Cylinders; Material-7075-T6 Aluminum Alloy



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

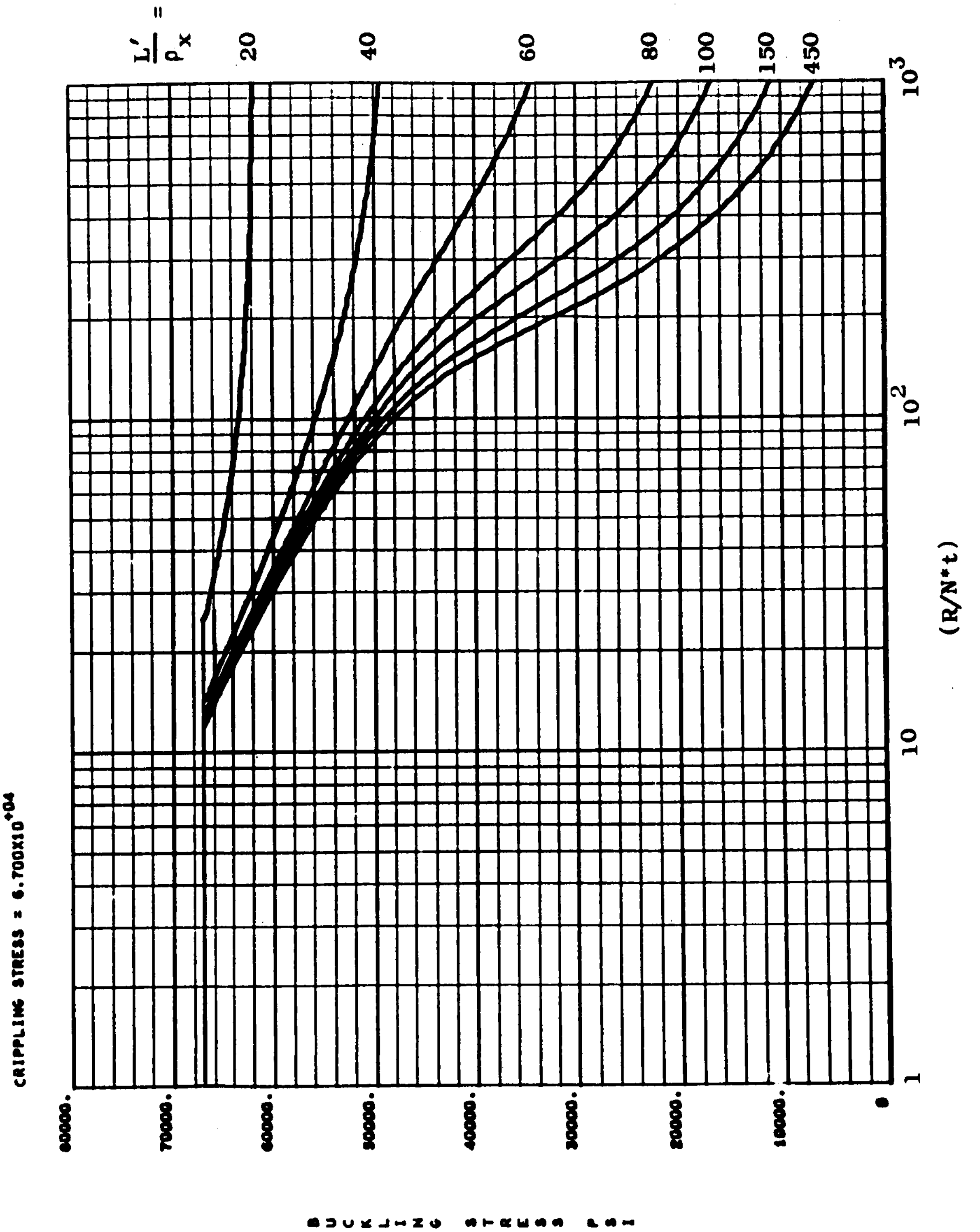
MATERIAL – 7075-T6 ALUMINUM ALLOY

Figure 4(a)



COMPRESSIVE BUCKLING STRESS FOR
LONGITUDINALLY STIFFENED CYLINDERS
MATERIAL - 7075-T6 ALUMINUM ALLOY

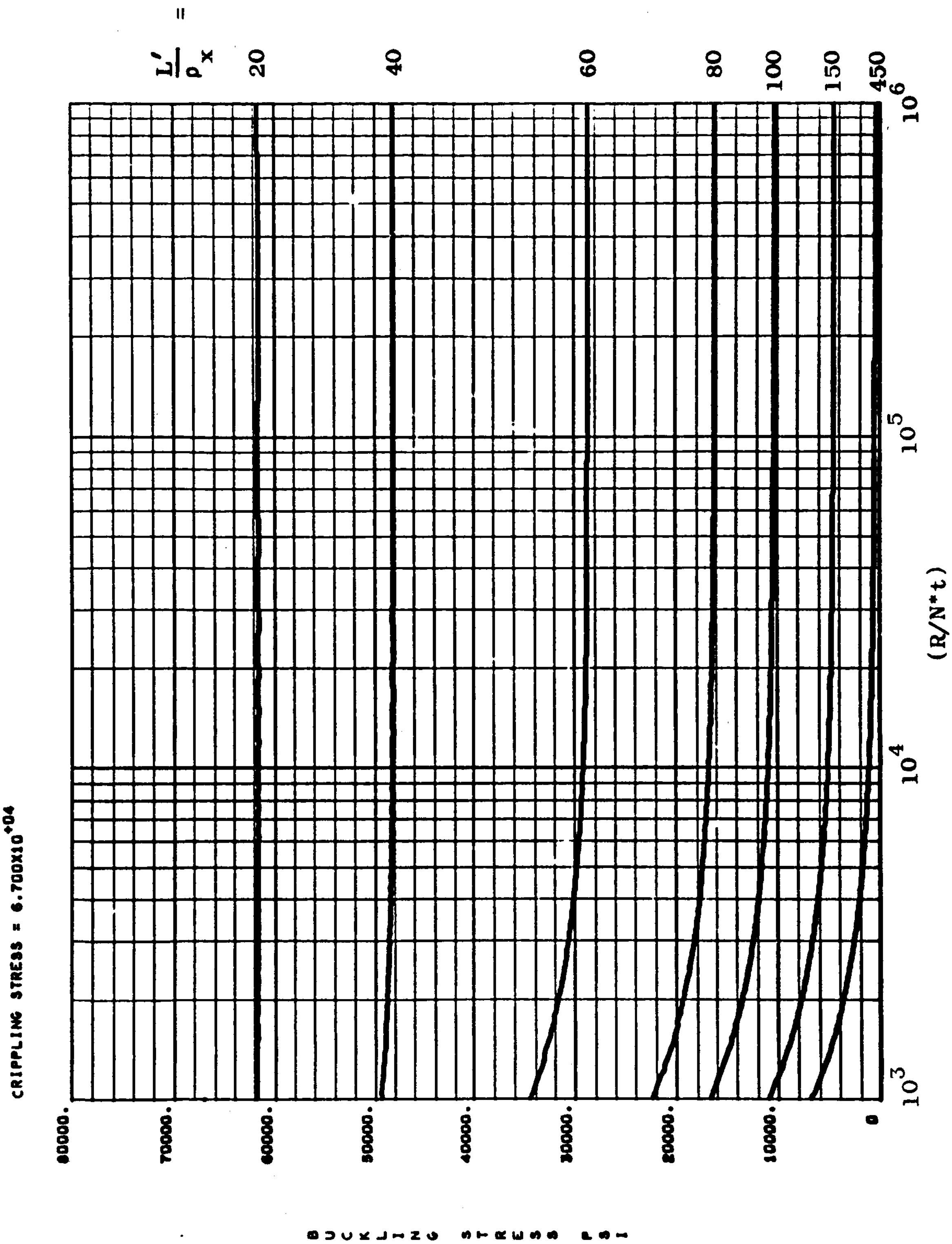
Figure 4(b)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

MATERIAL - 7075-T6 ALUMINUM ALLOY

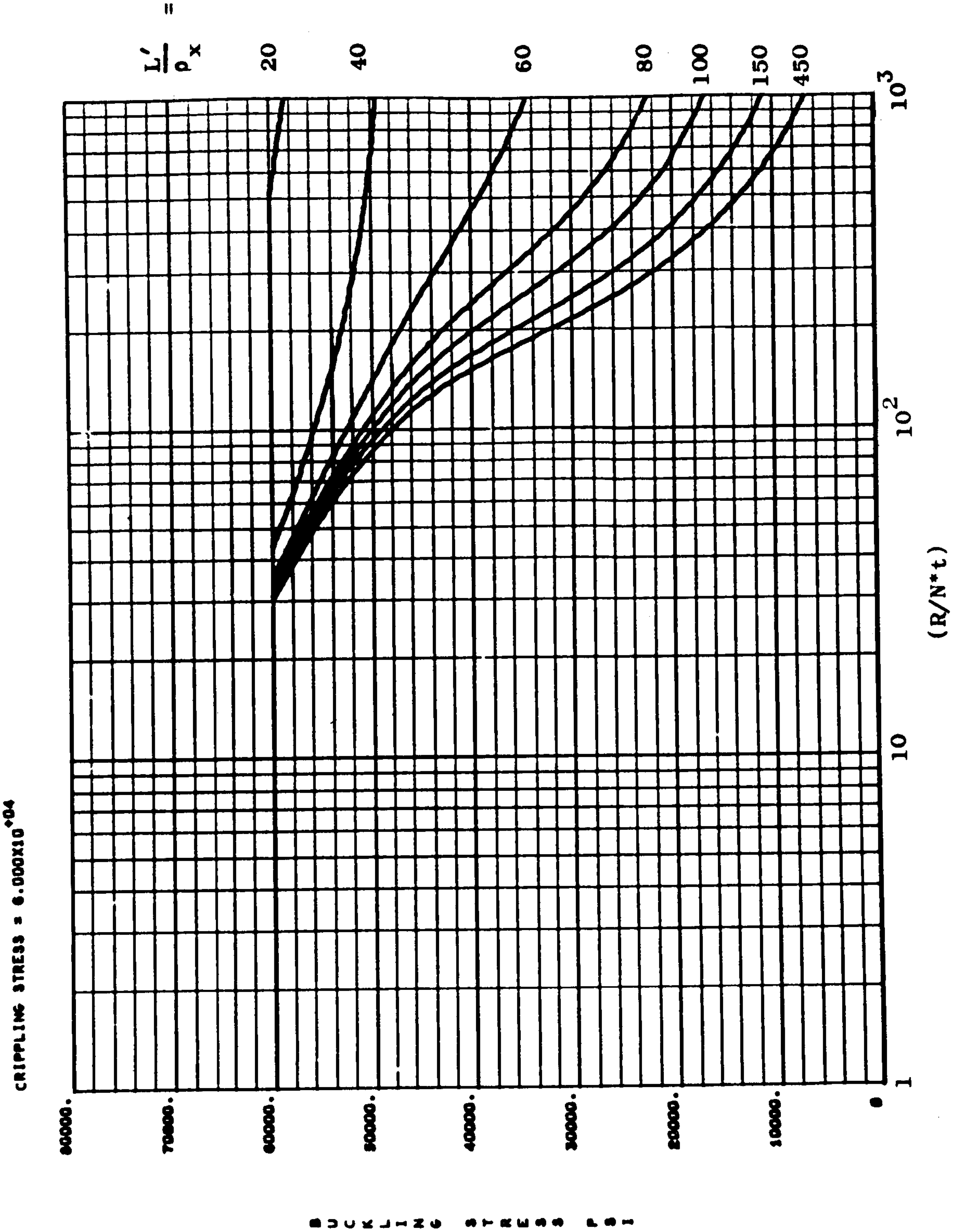
Figure 4(c)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

MATERIAL - 7075-T6 ALUMINUM ALLOY

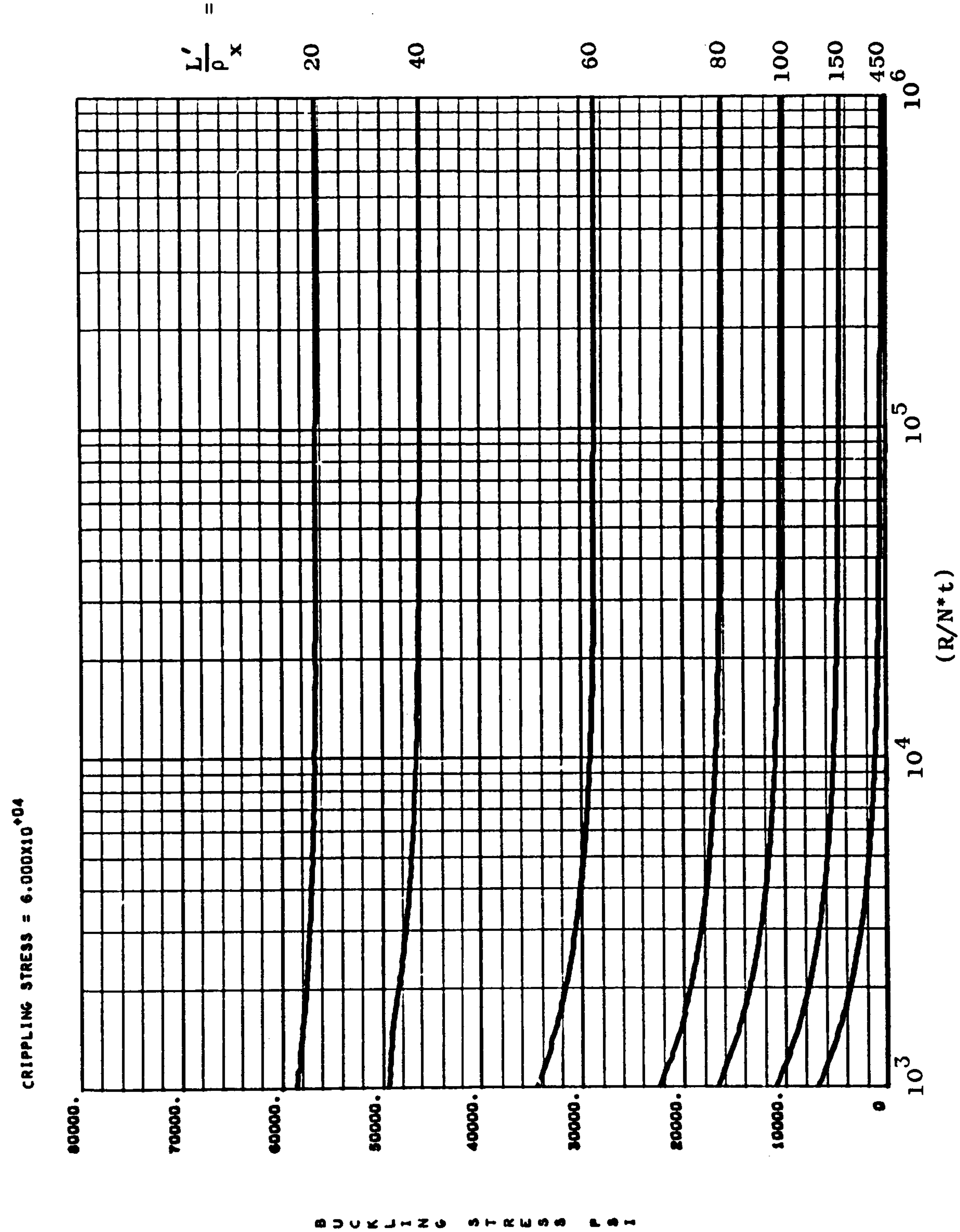
Figure 4(d)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

MATERIAL - 7075-T6 ALUMINUM ALLOY

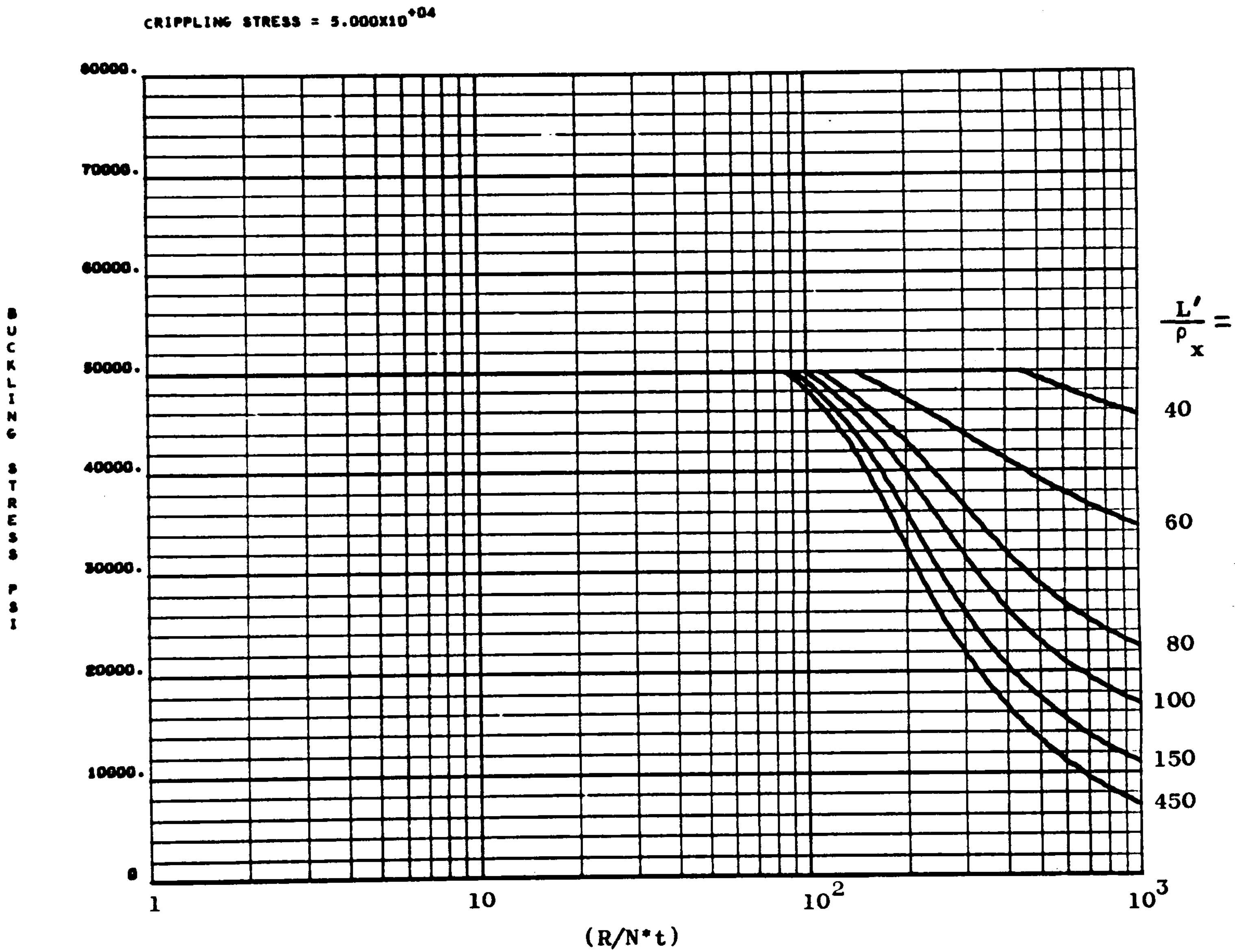
Figure 4(e)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

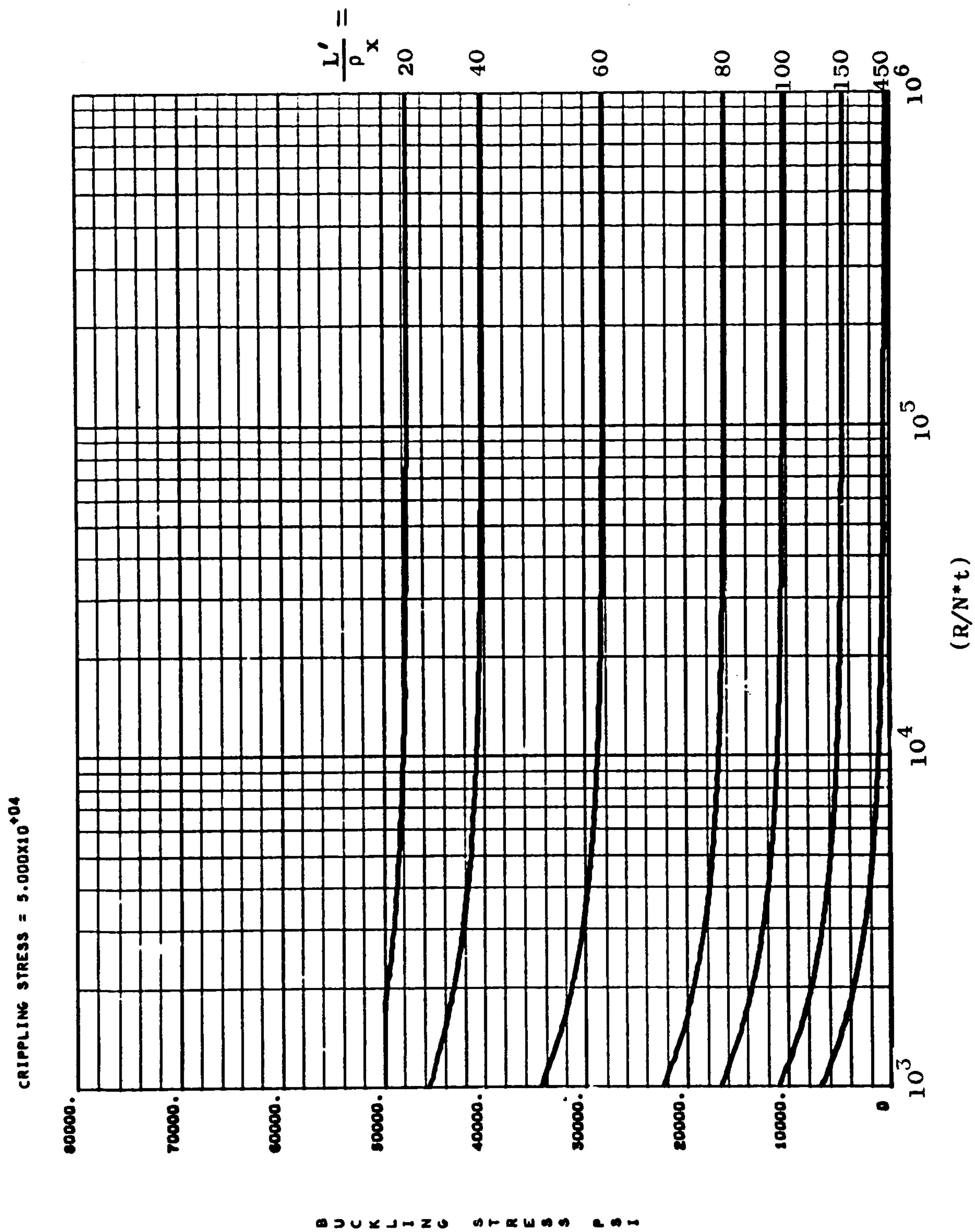
MATERIAL - 7075-T6 ALUMINUM ALLOY

Figure 4(f)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS
MATERIAL - 7075-T6 ALUMINUM ALLOY

Figure 4(g)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

MATERIAL - 7075-T6 ALUMINUM ALLOY

Figure 1(h)

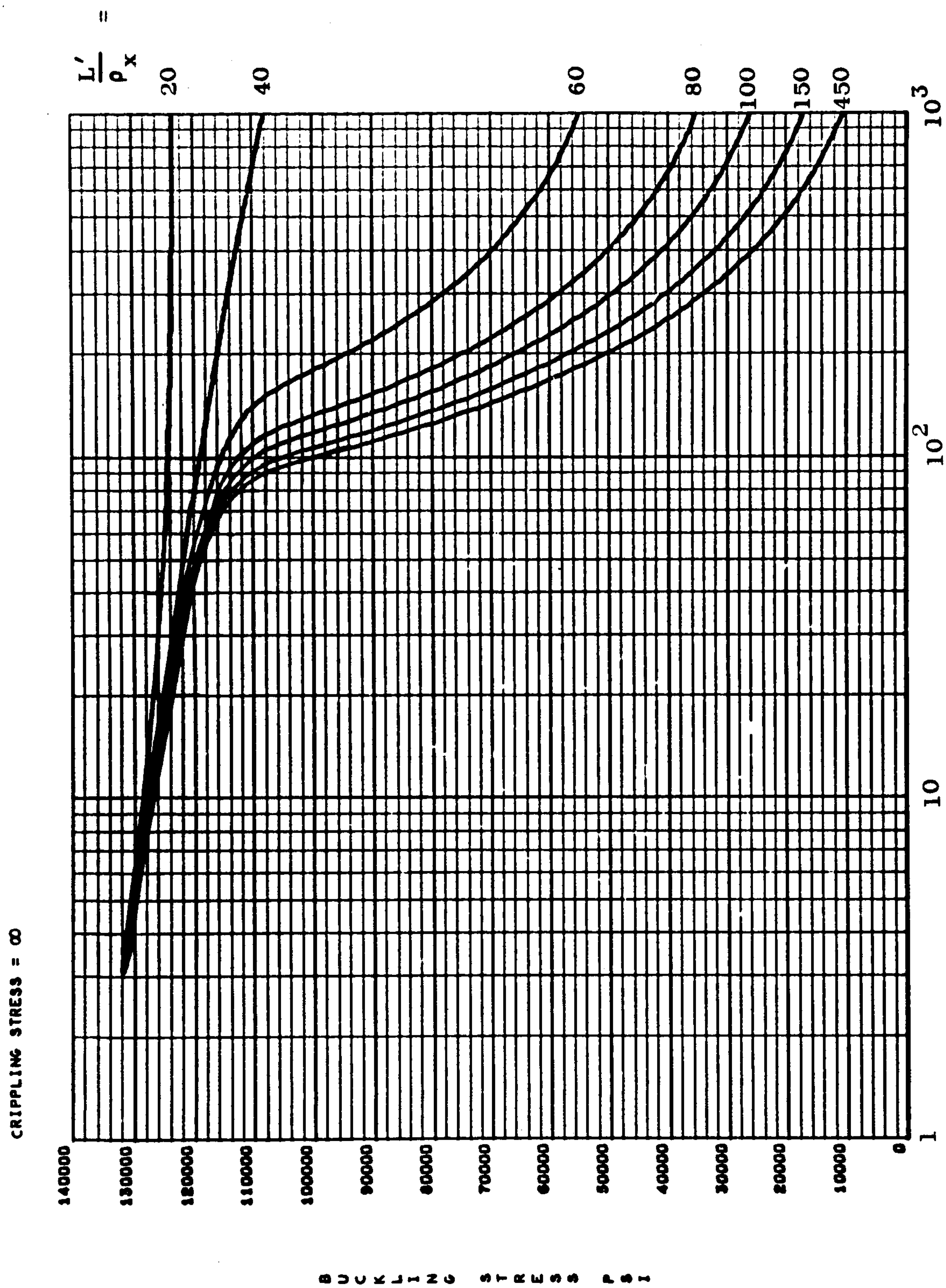
5.1.2 6Al-4V TITANIUM ALLOY (Annealed)

Table VII lists the families provided here for longitudinally stiffened cylinders made of annealed 6Al-4V titanium alloy. These curves are based upon the following values for the indicated material properties:

	E	$= 16.4 \times 10^6$ psi
	ν	$= .30$
	σ_{cy}	$= 132{,}000$ psi
Ramberg-Osgood	n	$= 35$
Ramberg-Osgood	$\sigma_{.7}$	$= 133{,}500$ psi

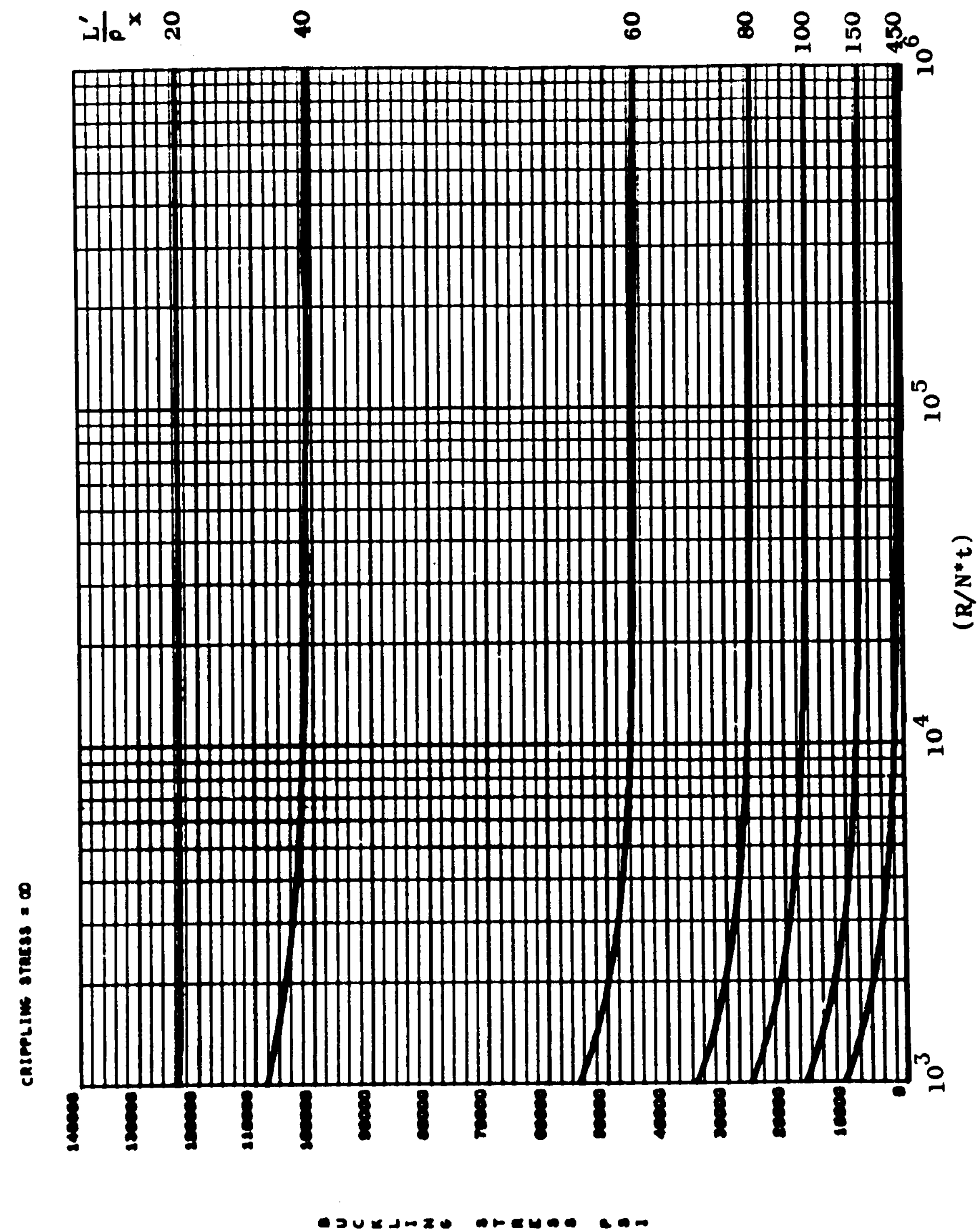
TABLE VII - Table of Contents for Curves of Compressive Buckling Stress for Longitudinally Stiffened Cylinders; Material - 6Al-4V Titanium Alloy (Annealed)

Figure Number	Crippling Stress, σ_{cc}	Range of $\left(\frac{R}{N^* t}\right)$	Page
5(a)	∞	$1 - 10^3$	5-15
5(b)	∞	$10^3 - 10^6$	5-16
5(c)	132,000	$1 - 10^3$	5-17
5(d)	132,000	$10^3 - 10^6$	5-18
5(e)	110,000	$1 - 10^3$	5-19
5(f)	110,000	$10^3 - 10^6$	5-20
5(g)	90,000	$1 - 10^3$	5-21
5(h)	90,000	$10^3 - 10^6$	5-22



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS
MATERIAL - 6Al-4V TITANIUM ALLOY (ANNEALED)

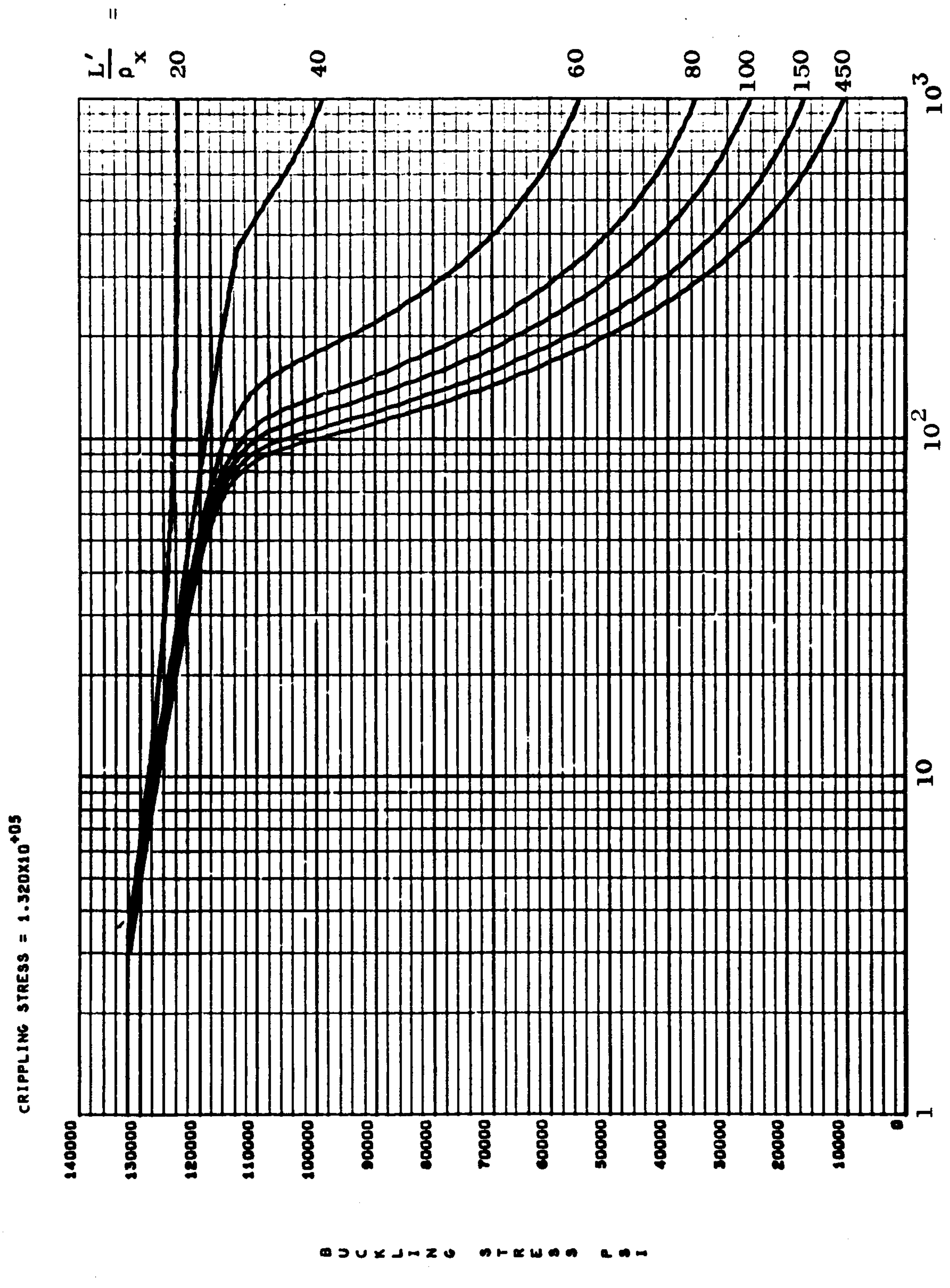
Figure 5(a)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

MATERIAL - 6Al-4V TITANIUM ALLOY (ANNEALED)

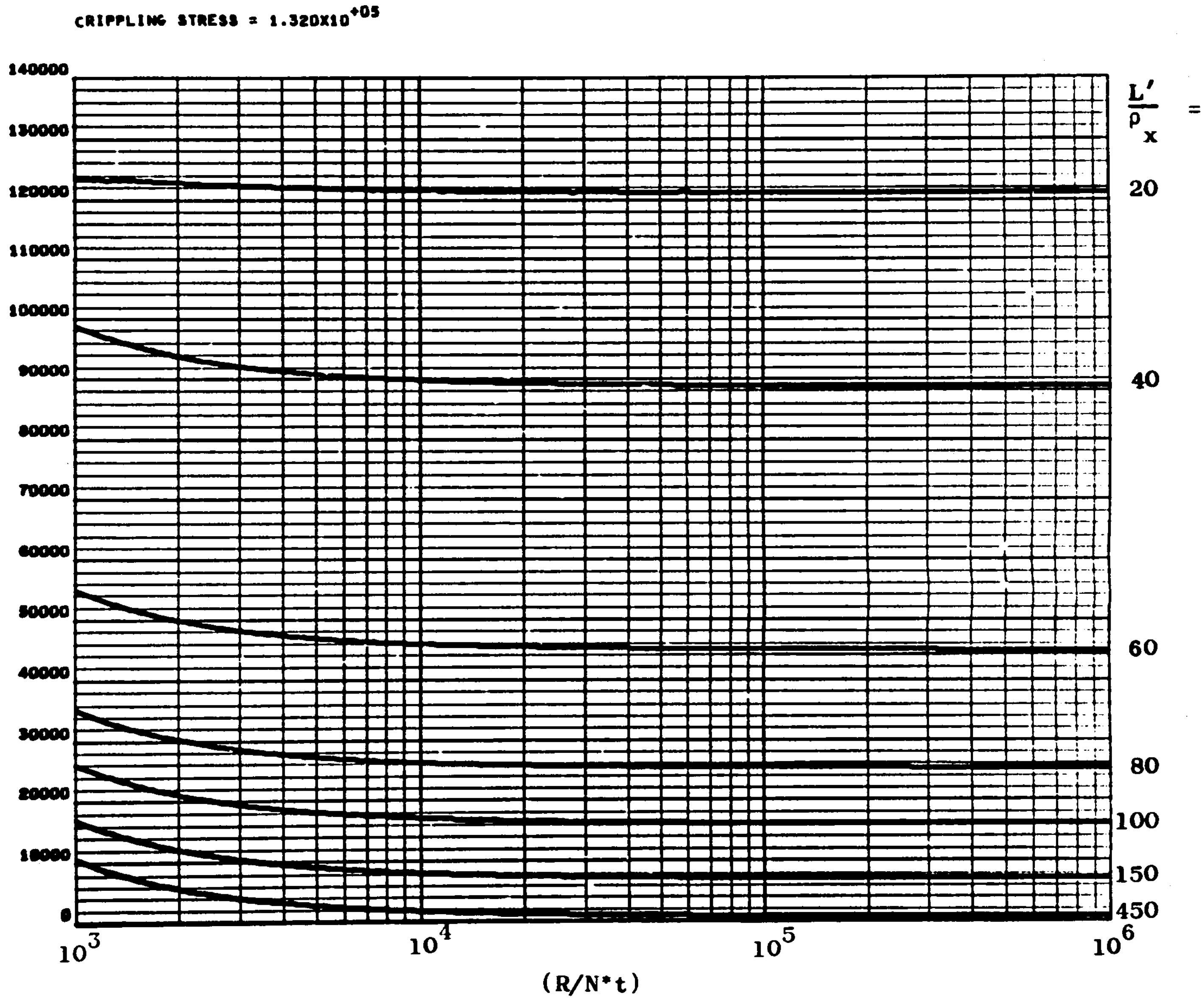
Figure 5(b)



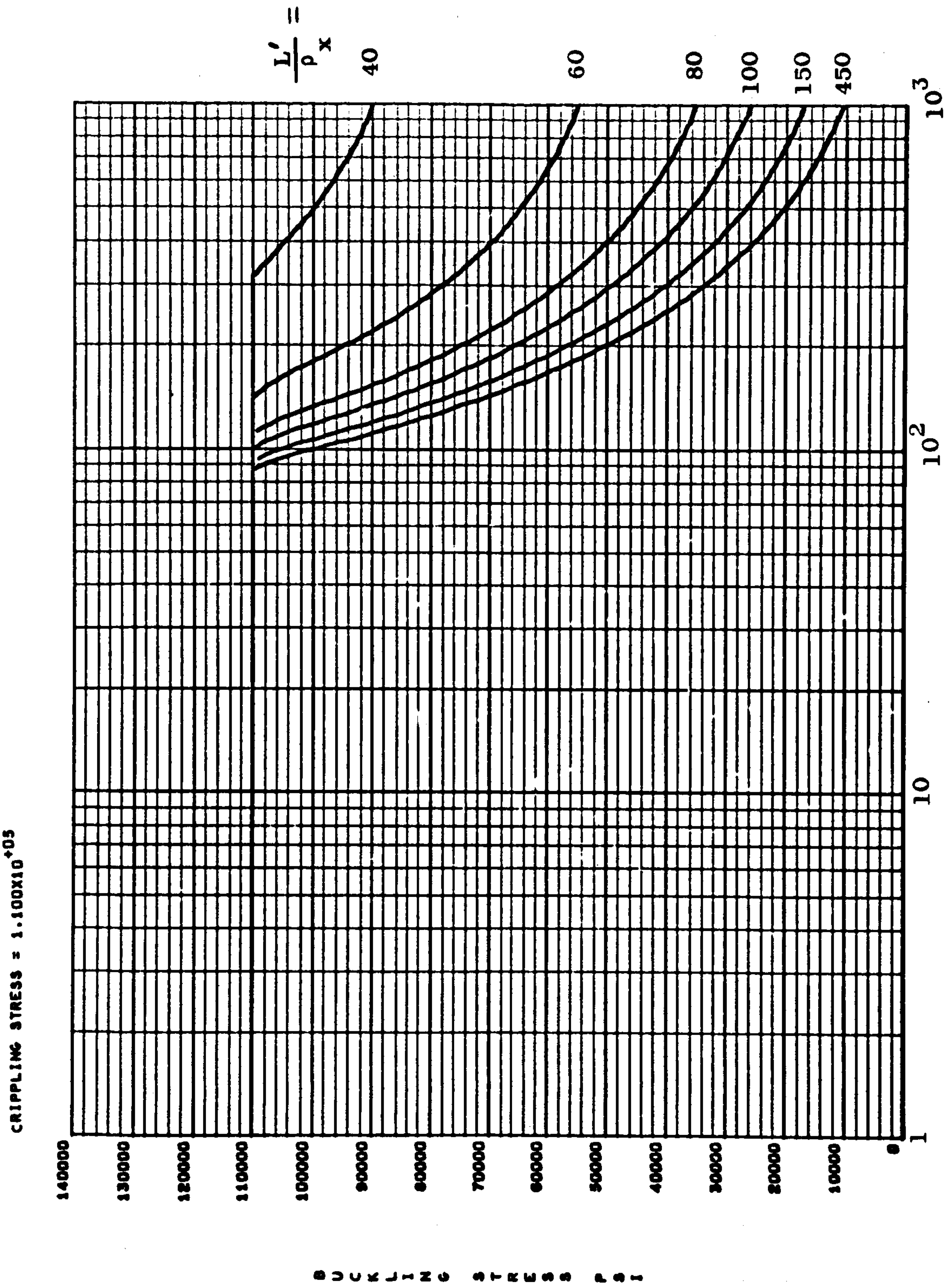
COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

MATERIAL - 6Al-4V TITANIUM ALLOY (ANNEALED)

Figure 5(c)

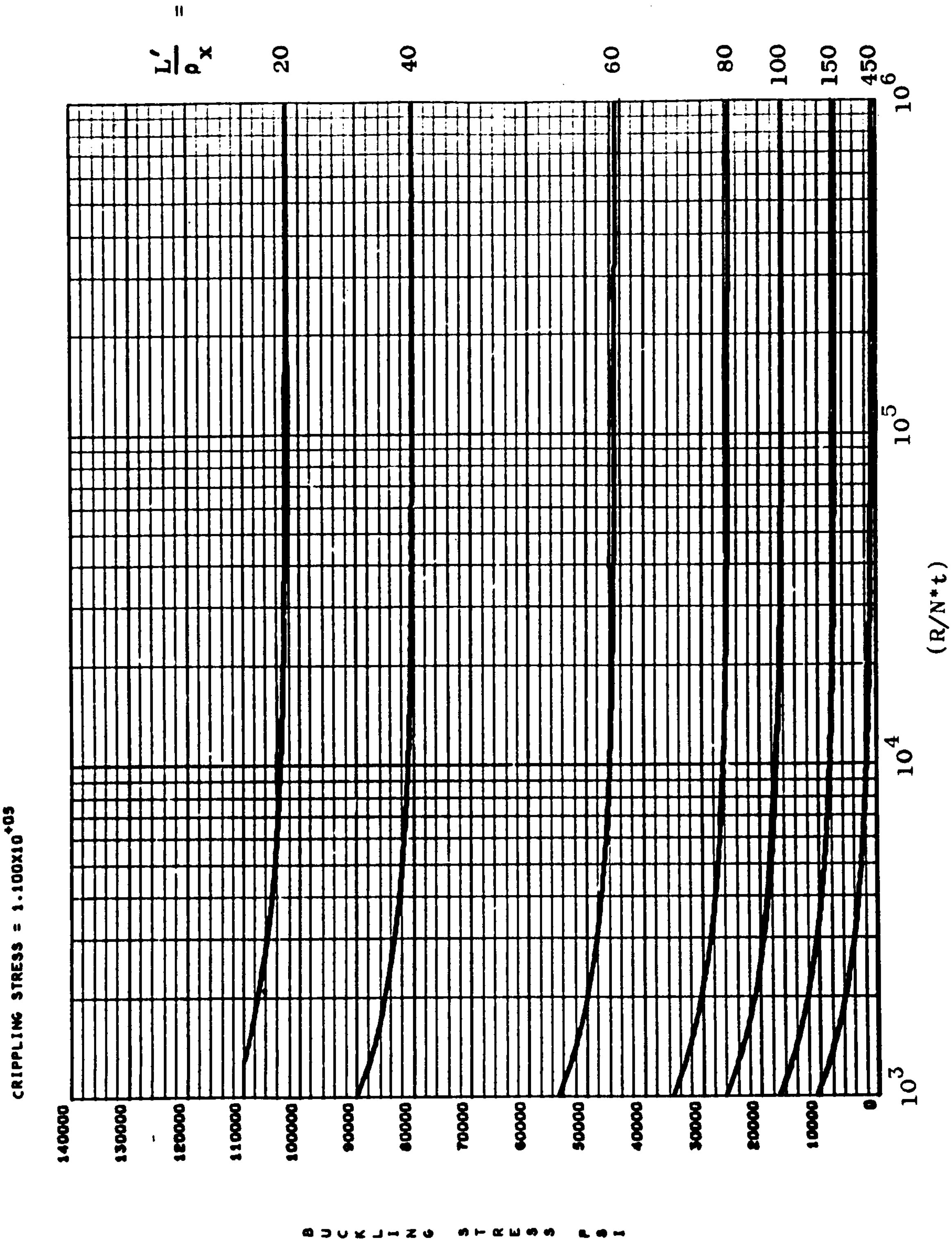


COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS
MATERIAL - 6Al-4V TITANIUM ALLOY (ANNEALED)
Figure 5(d)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS MATERIAL - 6Al-4V TITANIUM ALLOY (ANNEALED)

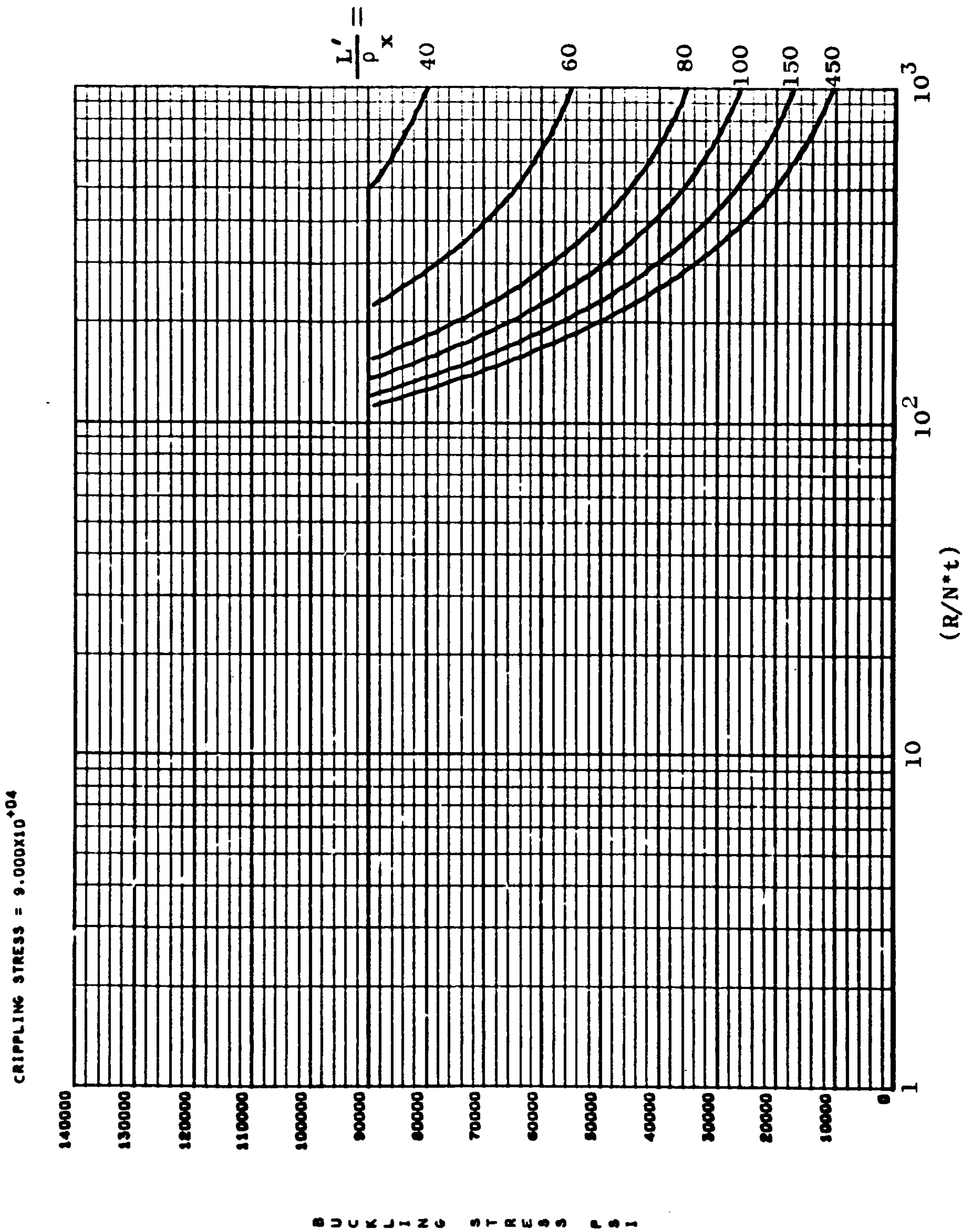
Figure 5(e)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

MATERIAL - 6Al-4V TITANIUM ALLOY (ANNEALED)

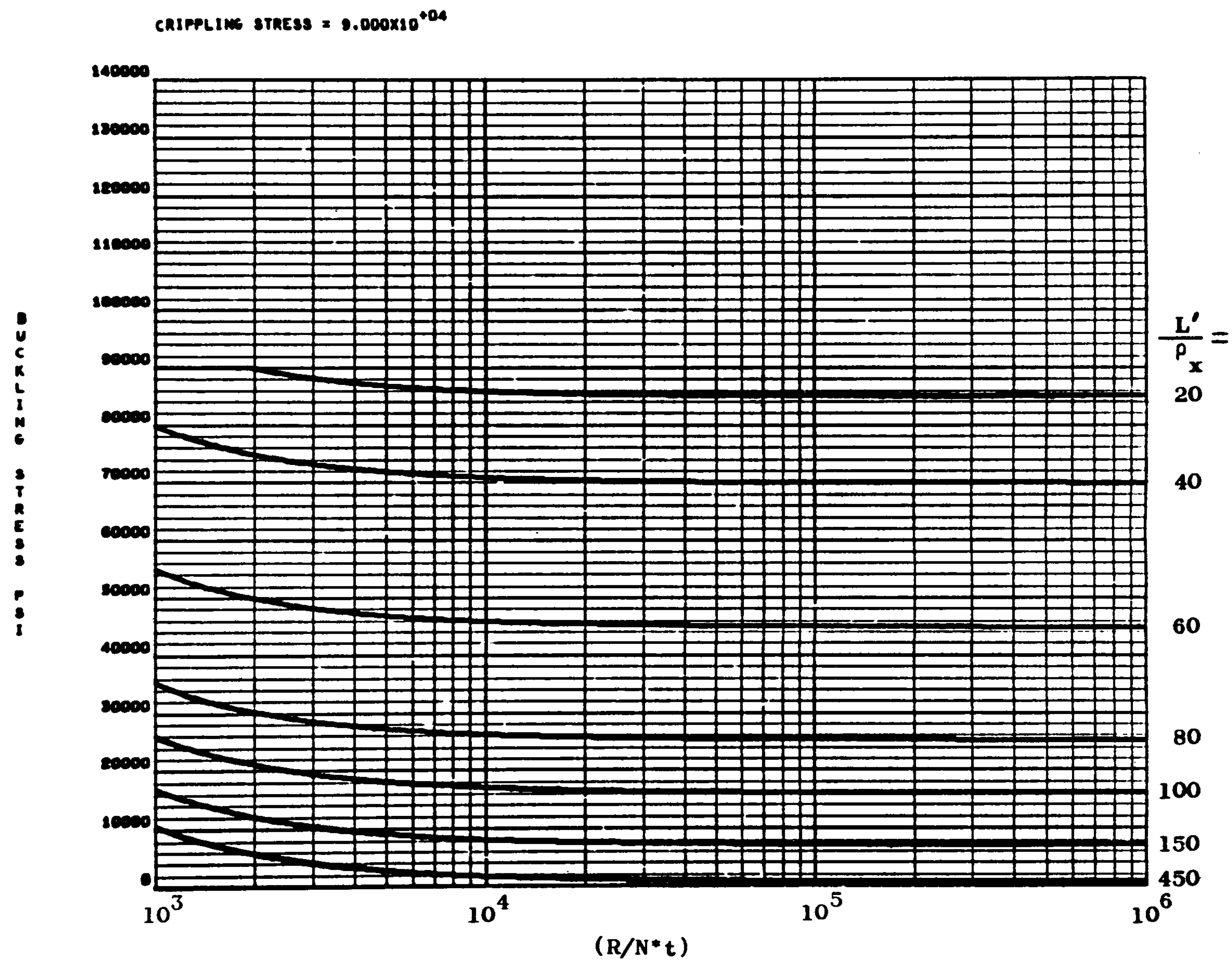
Figure 5(f)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

MATERIAL - 6Al-4V TITANIUM ALLOY (ANNEALED)

Figure 5(g)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

MATERIAL - 6Al-4V TITANIUM ALLOY (ANNEALED)

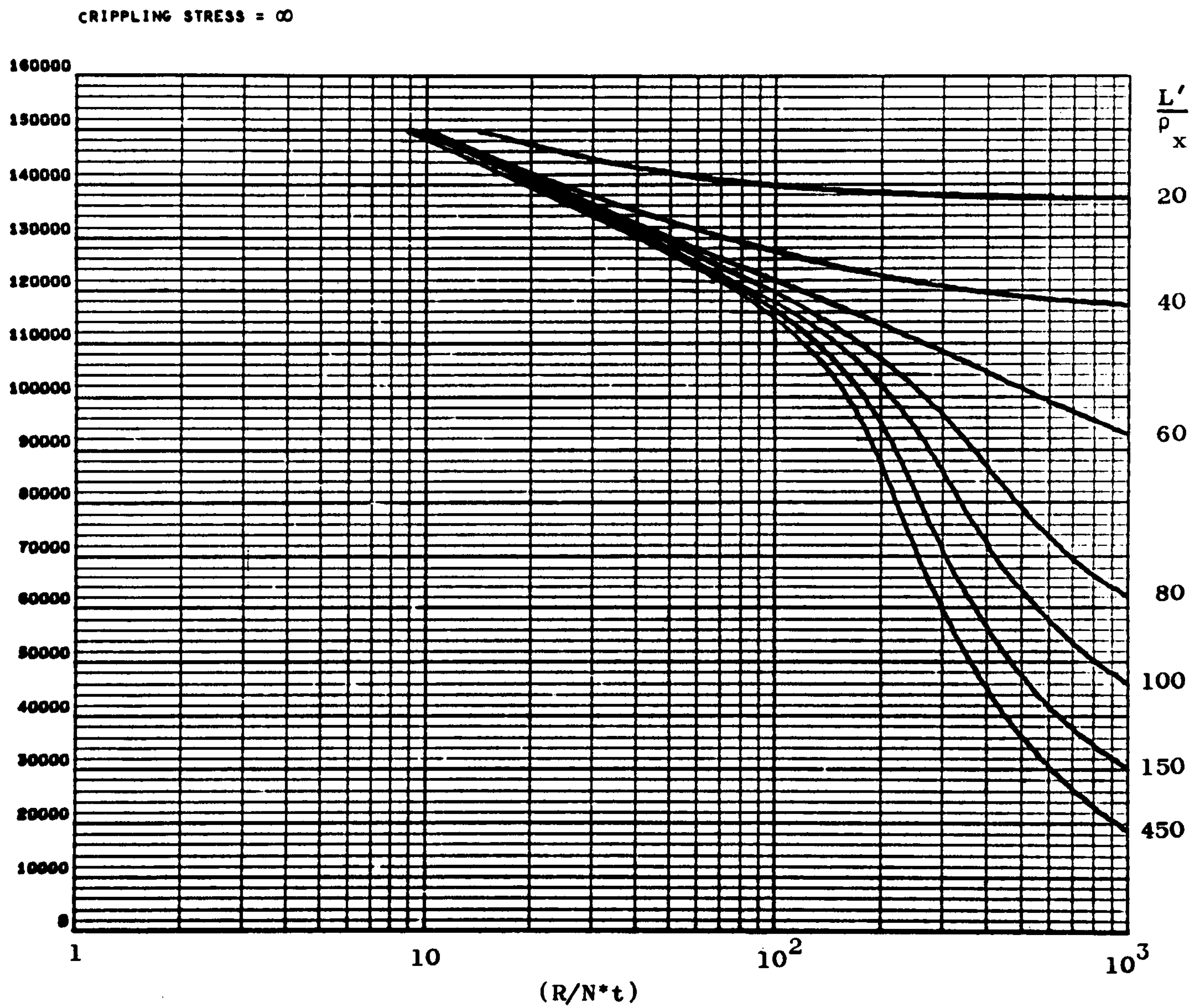
Figure 5(h)

5.1.3 718 NICKEL ALLOY (Annealed + Double Aged)

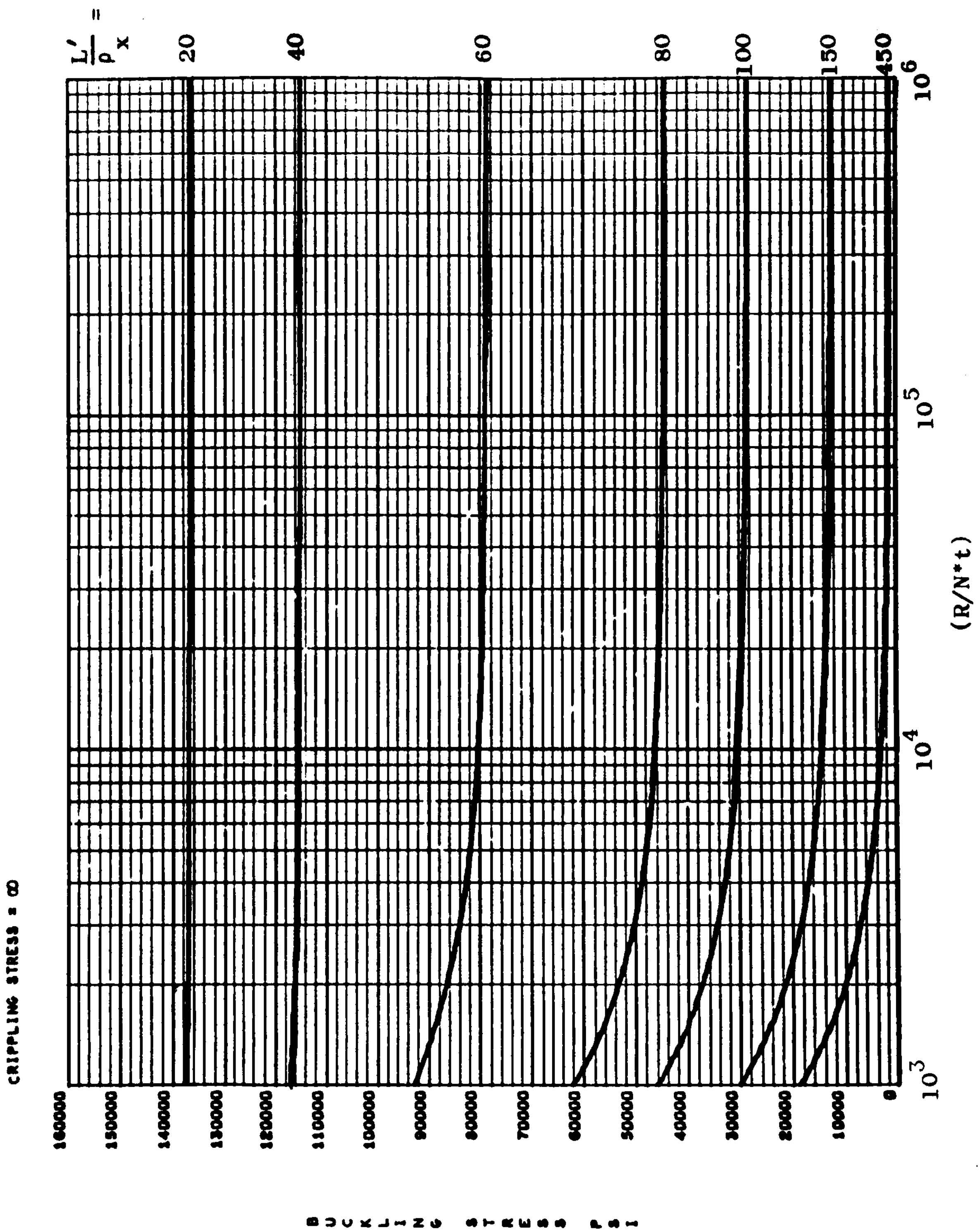
Table VIII lists the families provided here for longitudinally stiffened circular cylinders made of 718 nickel alloy (annealed + double aged). These curves are based upon the following values for the indicated material properties:

	E	$= 29.0 \times 10^6$ psi
	ν	$= .30$
	σ_{cy}	$= 150{,}000$ psi
Ramberg-Osgood	n	$= 12.7$
Ramberg-Osgood	$\sigma_{.7}$	$= 150{,}500$ psi

TABLE VIII - Table of Contents for Curves of Compressive Buckling Stress for Longitudinally Stiffened Cylinders; Material - 718 Nickel Alloy (Annealed + double aged)



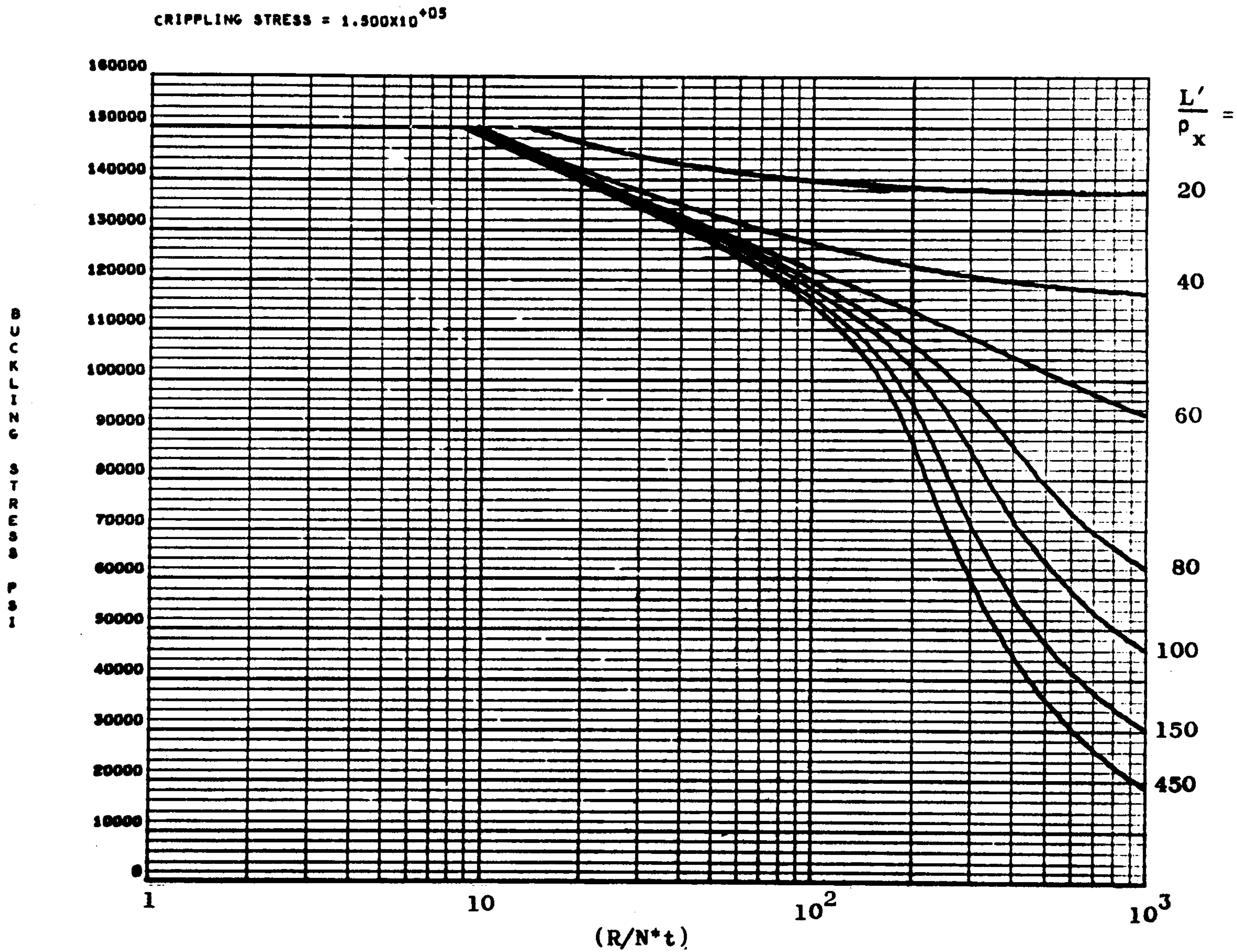
COMPRESSIVE BUCKLING STRESS FOR
LONGITUDINALLY STIFFENED CYLINDERS
MATERIAL - 718 NICKEL ALLOY (ANNEALED + DOUBLE AGED)
Figure 6(a)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

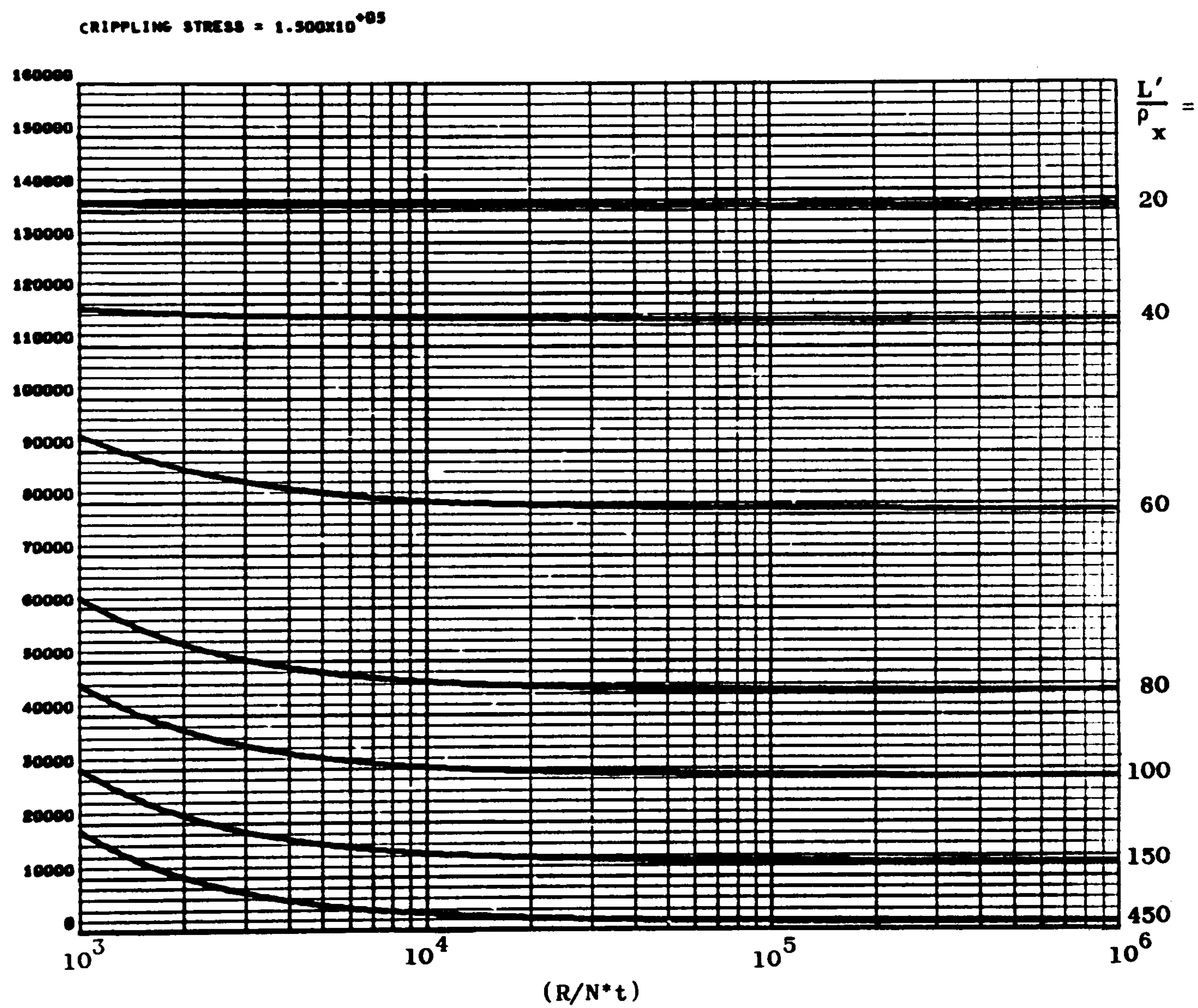
MATERIAL - 718 NICKEL ALLOY (ANNEALED + DOUBLE AGED)

Figure 6(b)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS
MATERIAL - 718 NICKEL ALLOY (ANNEALED + DOUBLE AGED)

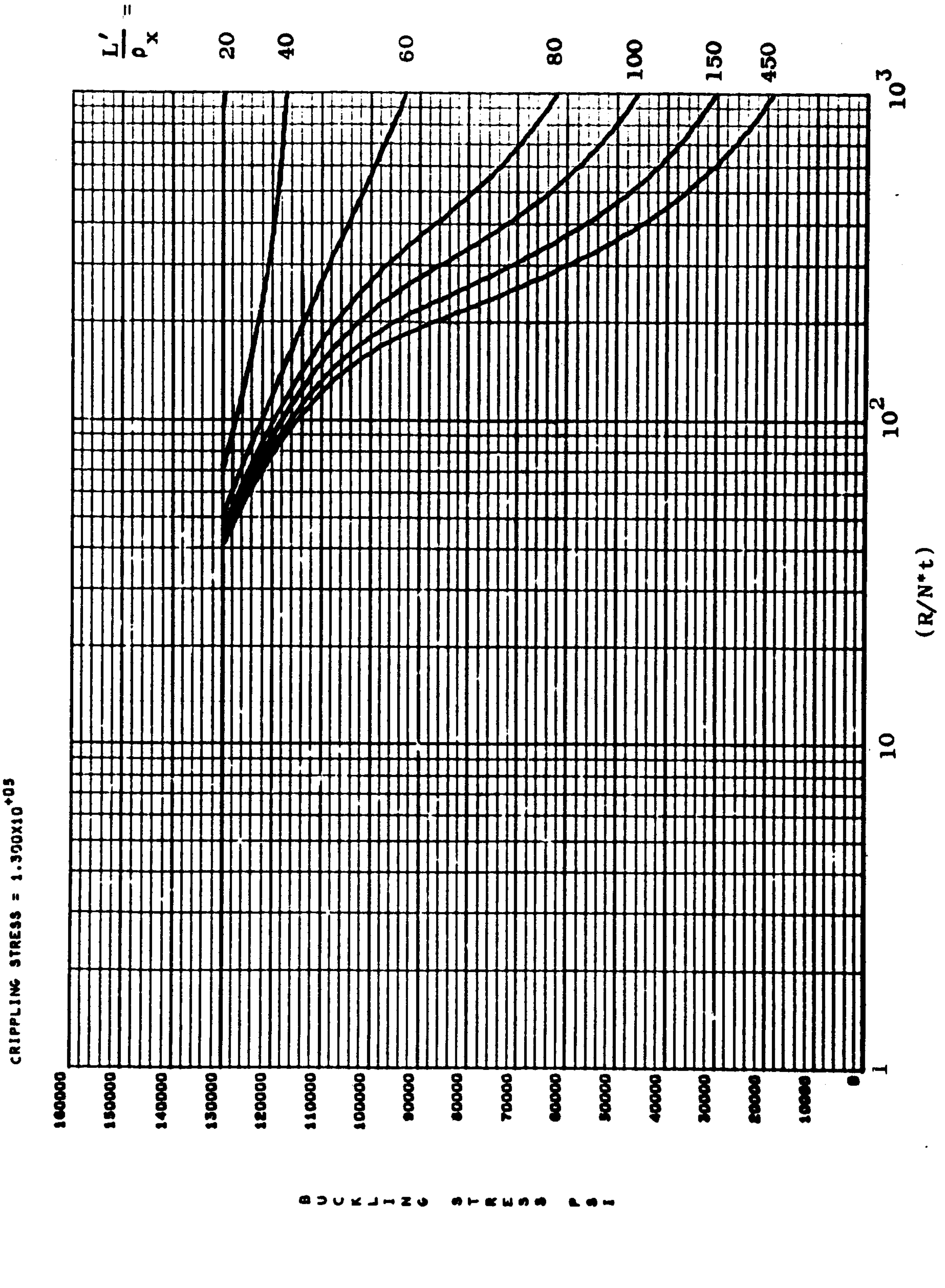
Figure 6(c)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

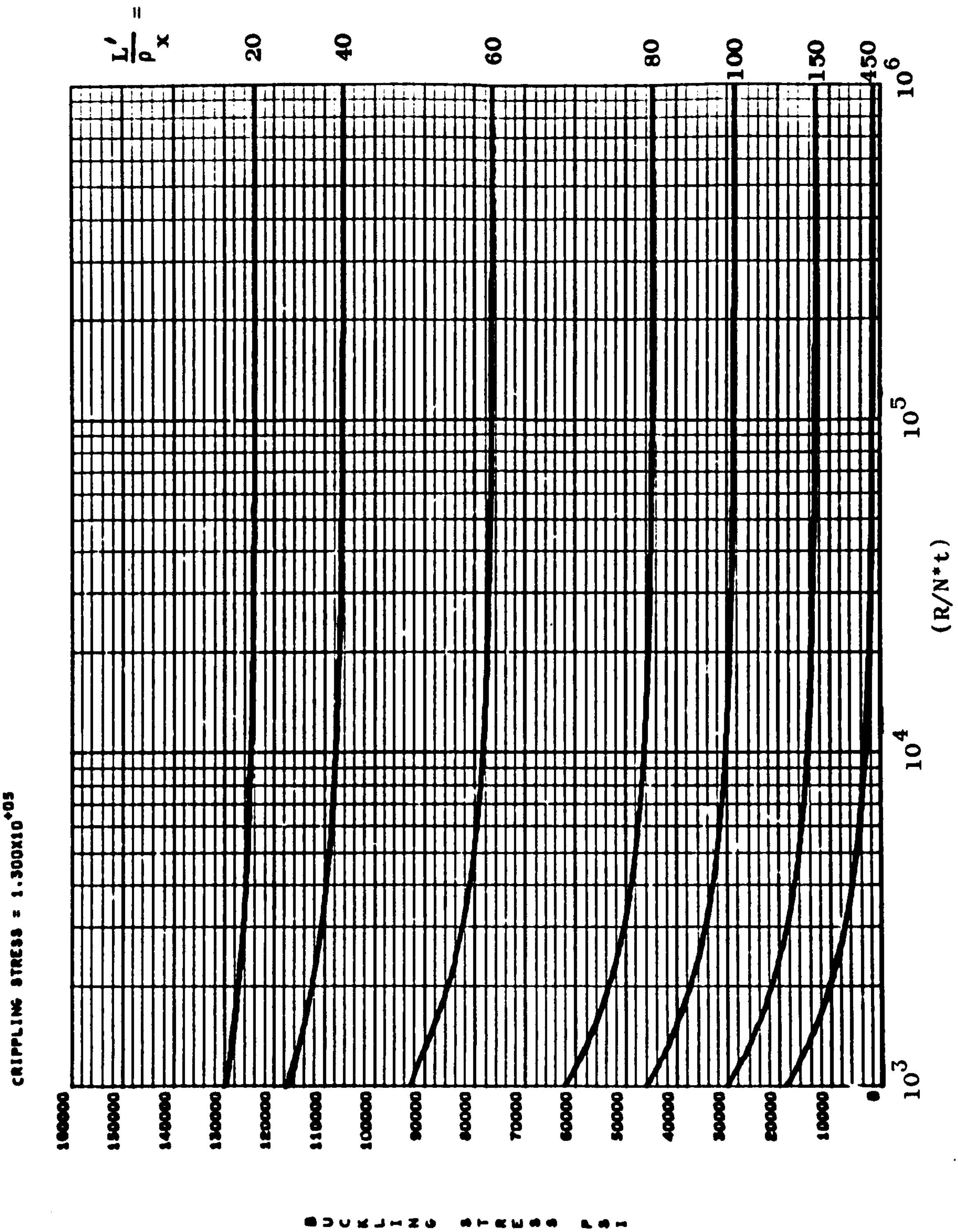
MATERIAL - 718 NICKEL ALLOY (**ANNEALED** + **DOUBLE** AGED)

Figure 6(d)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS MATERIAL -718 NICKEL ALLOY (ANNEALED + DOUBLE AGED)

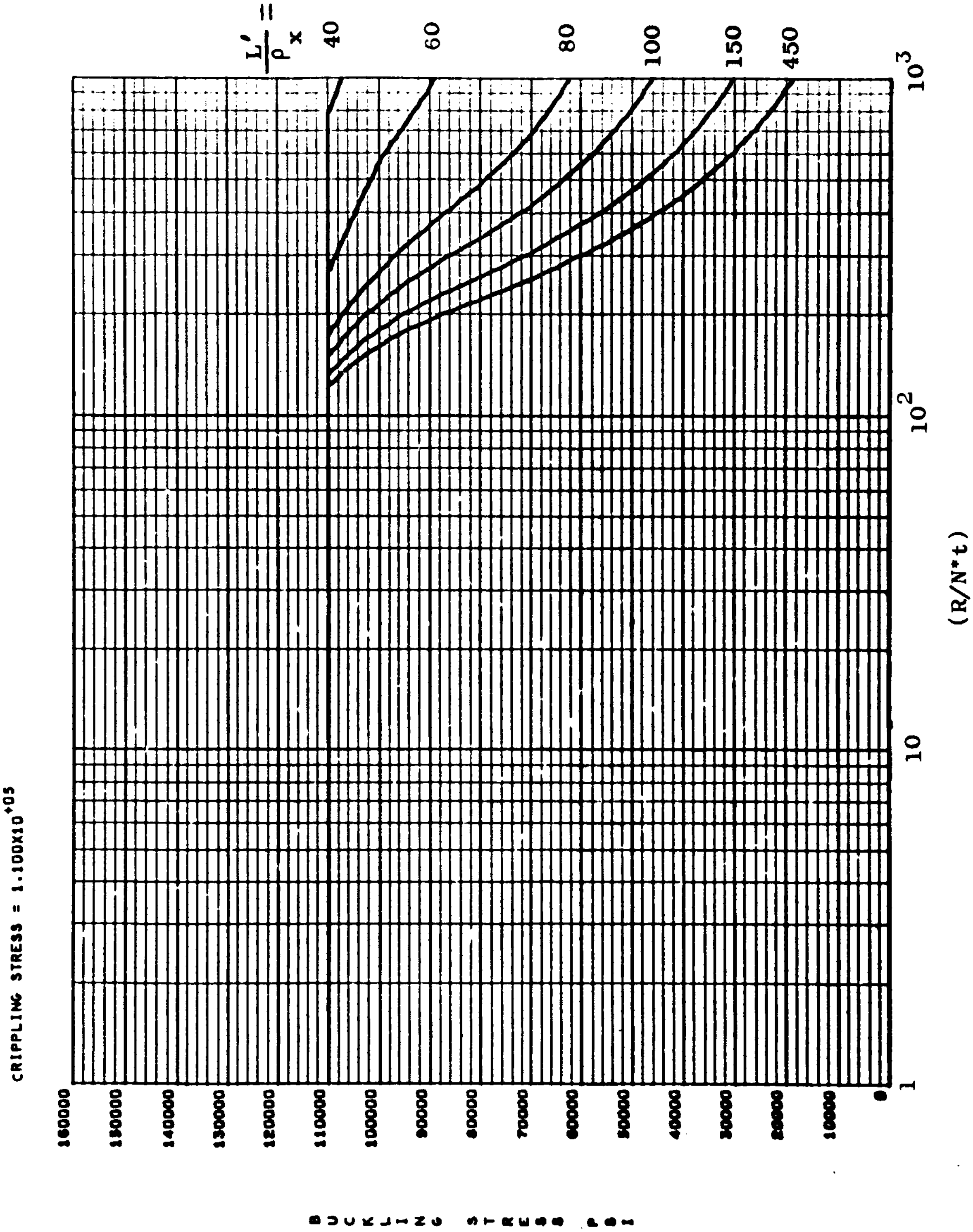
Figure 6(e)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

MATERIAL - 718 NICKEL ALLOY (ANNEALED + DOUBLE AGED)

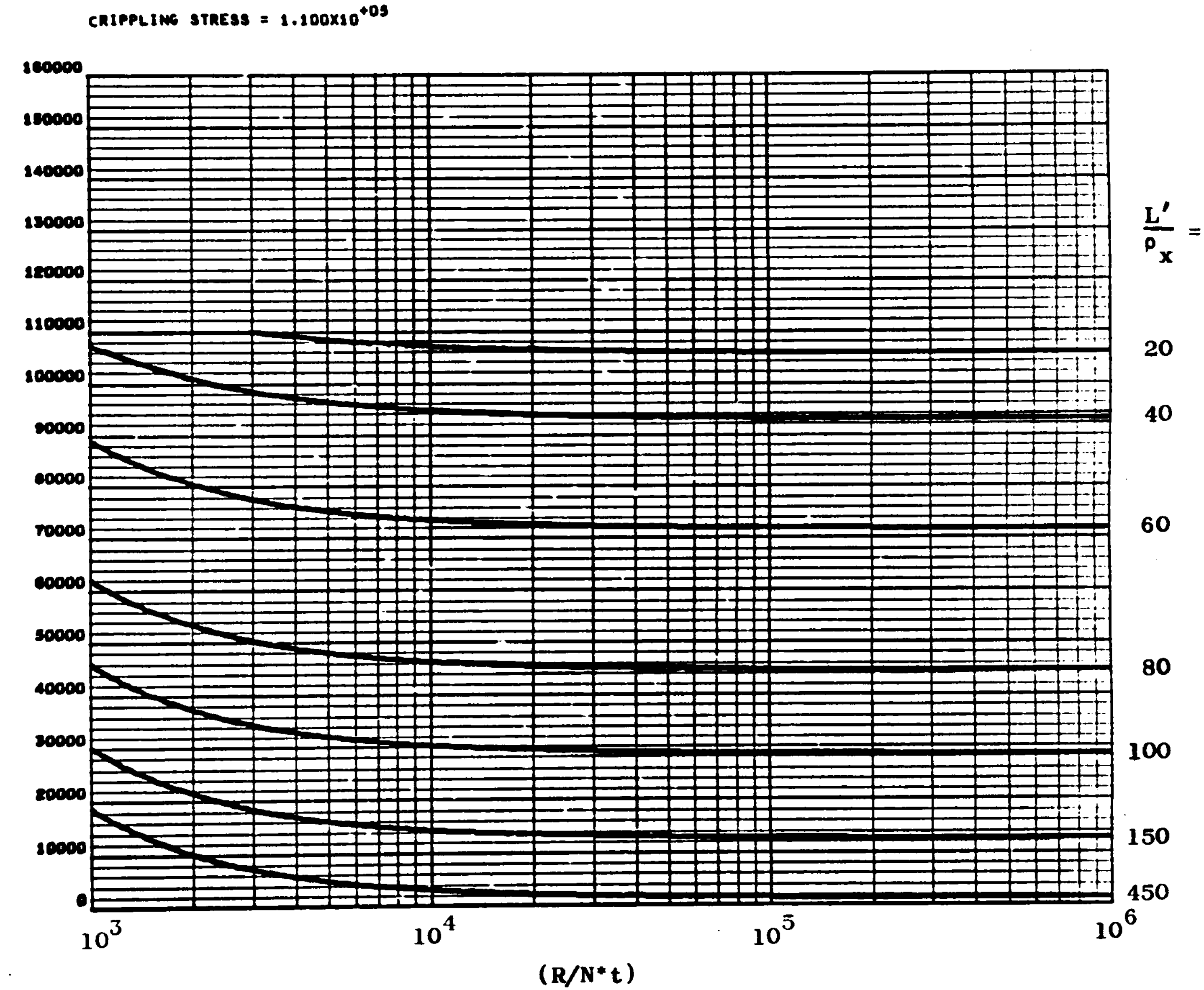
Figure 6(f)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

MATERIAL -718 NICKEL ALLOY (ANNEALED + DOUBLE AGED)

Figure 6(g)



COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED CYLINDERS

MATERIAL – 718 NICKEL ALLOY (ANNEALED + DOUBLE AGED)

Figure 6(h)

5.2 MINIMIZATION FACTOR N*

The curves of this section present values for a minimization factor N* [see equation (2-22)] used in the analysis of instability in longitudinally stiffened circular cylinders subjected to axial compression. To make proper use of these curves, one should refer to the instructions furnished in SECTION 4 "ANALYSIS METHOD". All of these curves were developed by using digital computer program 4235 (see SECTION 7) in conjunction with an automatic plotting machine. The machine located individual points through which the curves were drawn by hand. Since the plotting machine does not have the capability to print out lower case letters, the quantities t and z are denoted on the plots in upper case notation.

In applying the curves of Figure 7, one may interpolate between the given curves for constant (L'/R). However, due to the existence of inherent trend reversals, extrapolation beyond the (L'/R) range shown for any given family is prohibited. Note, for example, the trend reversal which occurs between the (L'/R) values of 0.30 and 1.2 for the case where $(\bar{t}_x/t) = 1.2$ and $(\bar{z}_x/R) = +0.05$.

The curves of Figure 7 involve the ratios (T BAR/T), (Z BAR/R), (L'/R), and (RADIUS/T), where

T BAR = $\bar{t}_x$	=	Thickness of appropriate smeared-out area of cross sections lying in planes normal to the axis of revolution (see notes following Table X). The subscript x does not appear on the plots since no confusion can result in the case of cylinders having only longitudinal stiffening.
T = t	=	Thickness of basic cylindrical skin in conventional skin-stringer constructions. Since this quantity enters into the equation for N* through more than one elastic constant, the curves of Figure 7 cannot generally be applied

to other types of configurations (corrugations, for example). To obtain N* for these other constructions, one may use digital computer program 4235.

Z BAR = $\bar{z}_x$ = Eccentricity defined in Table X. The subscript x does not appear on the plots since no confusion can result in the case of cylinders having only longitudinal stiffening. This quantity is positive for internally stiffened cylinders and is negative for externally stiffened cylinders. In addition, $\bar{z}$ = 0 when the stiffeners are symmetrical about the reference surface (the middle surface of the basic cylindrical skin).

RADIUS = R = Radius to middle surface of basic cylindrical skin.

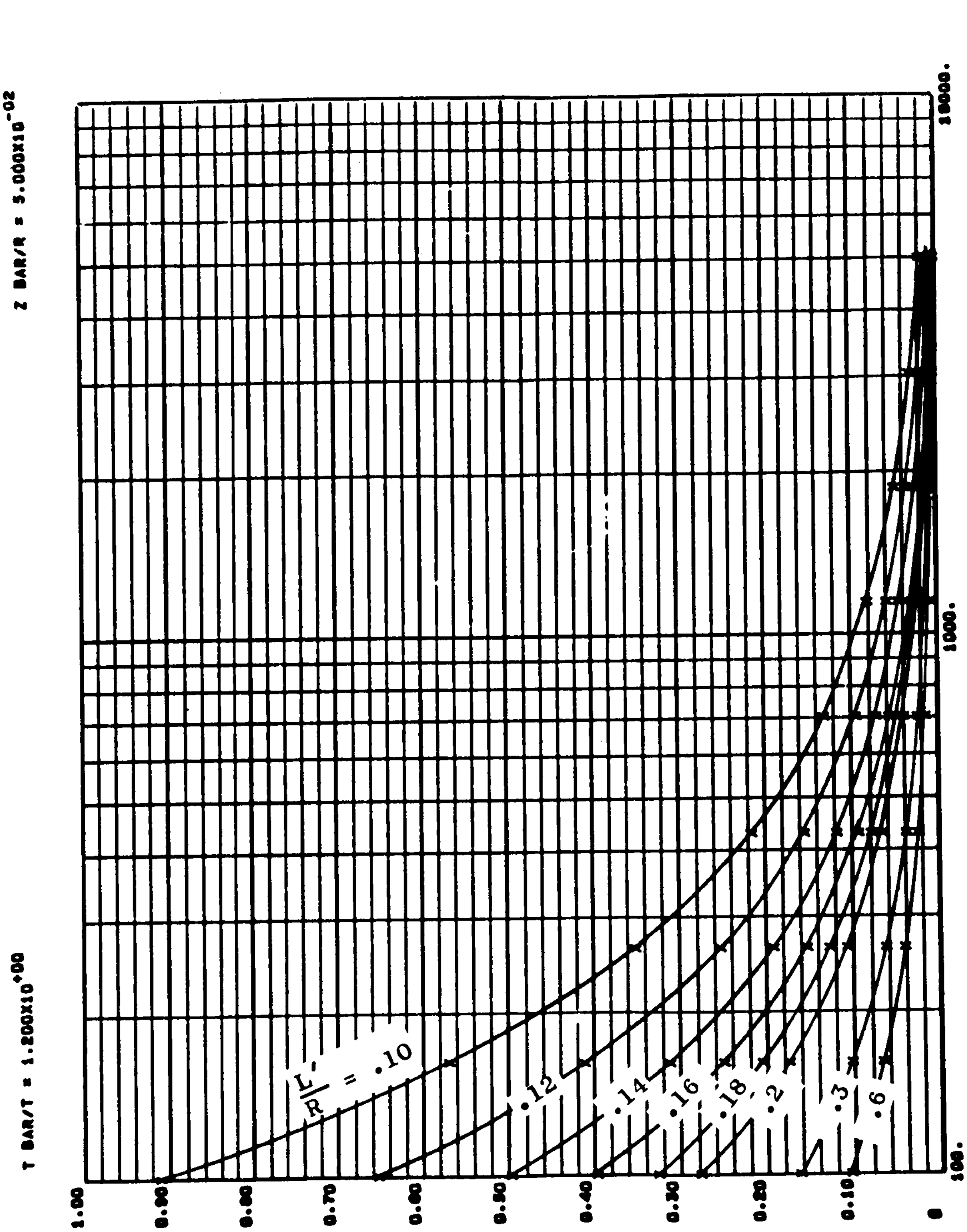
L' = An effective length which, for short cylinders, may be computed from the equation

$$L' = \frac{L}{\sqrt{C_F}} \tag{5-3}$$

The quantity C_F is the fixity coefficient which would apply to a wide-column having the same boundary conditions as the actual cylinder. See SECTION 4 concerning certain checks which should be made in connection with the L′ value.

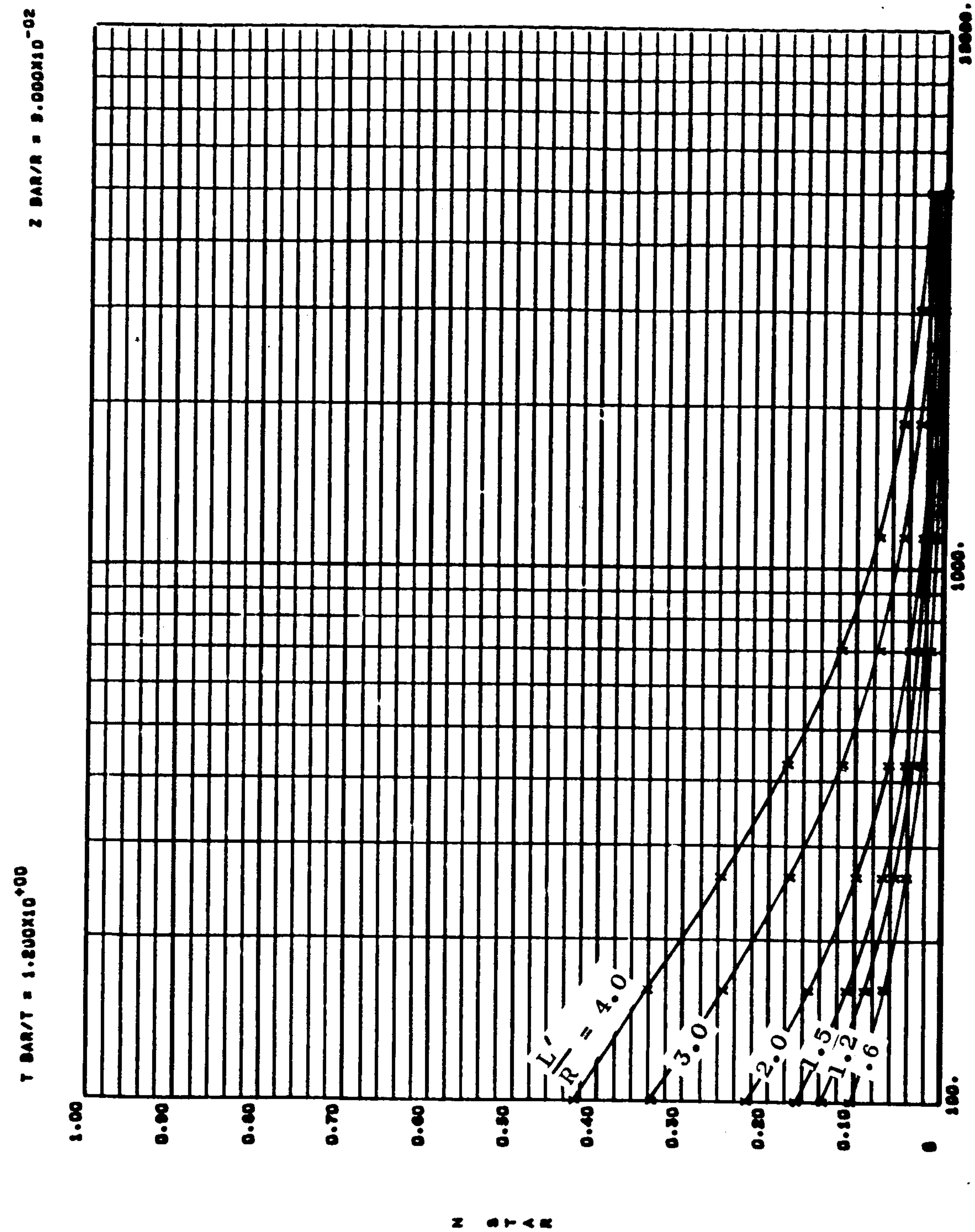
Table IX lists the families provided here.

TABLE IX - Table of Contents for Curves of the Minimization Factor N* for Longitudinally Stiffened Circular Cylinders



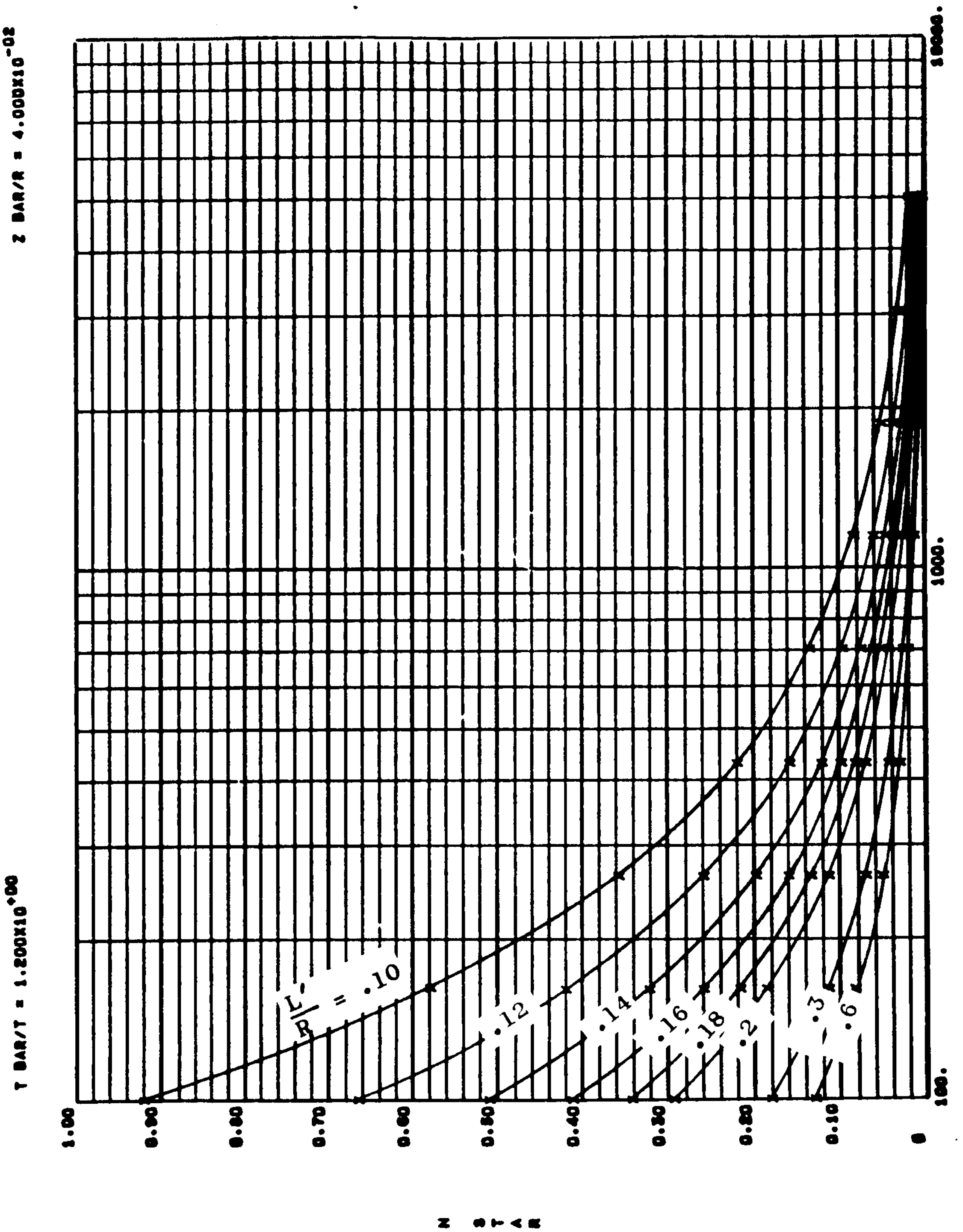
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(a)



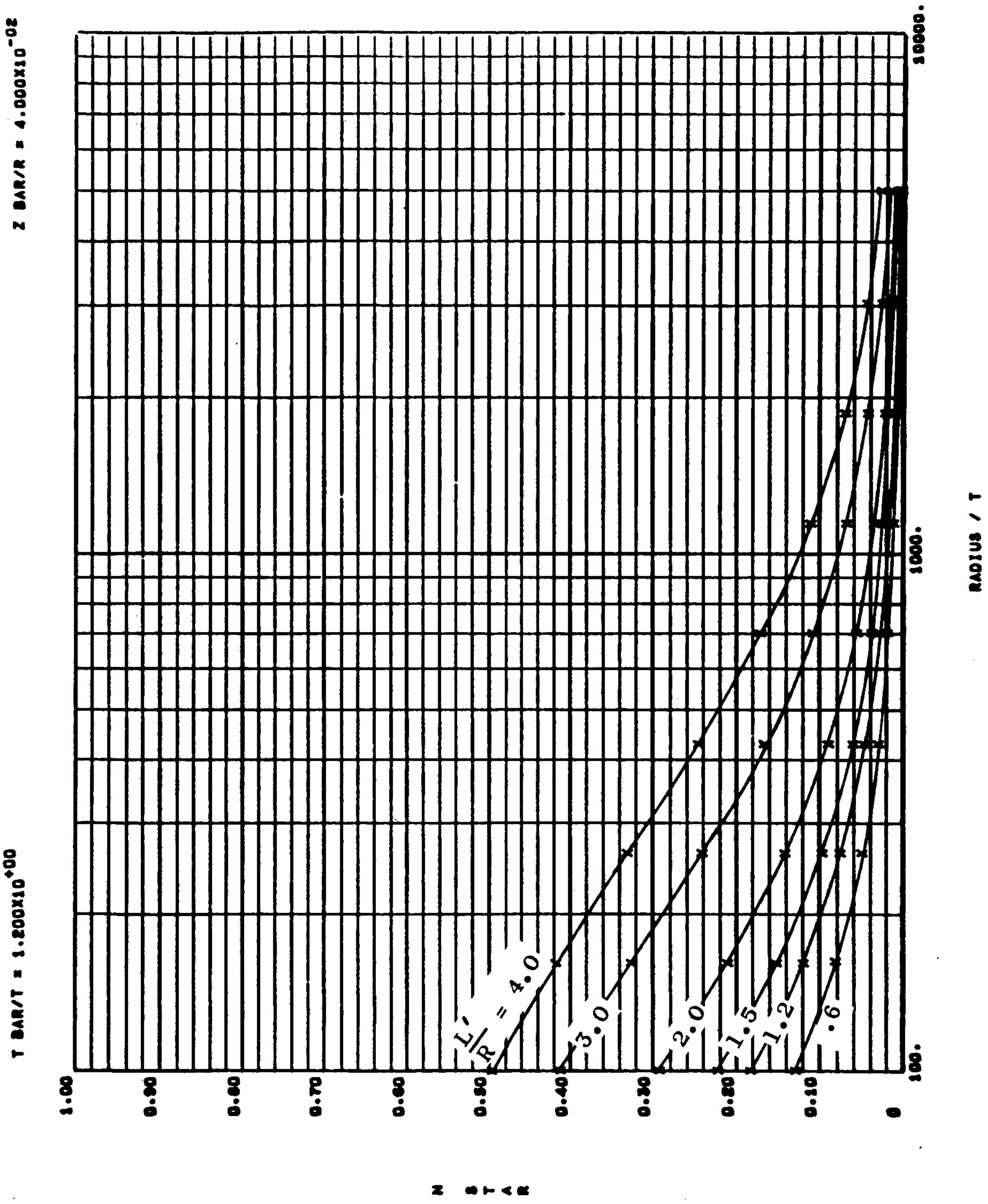
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(a)



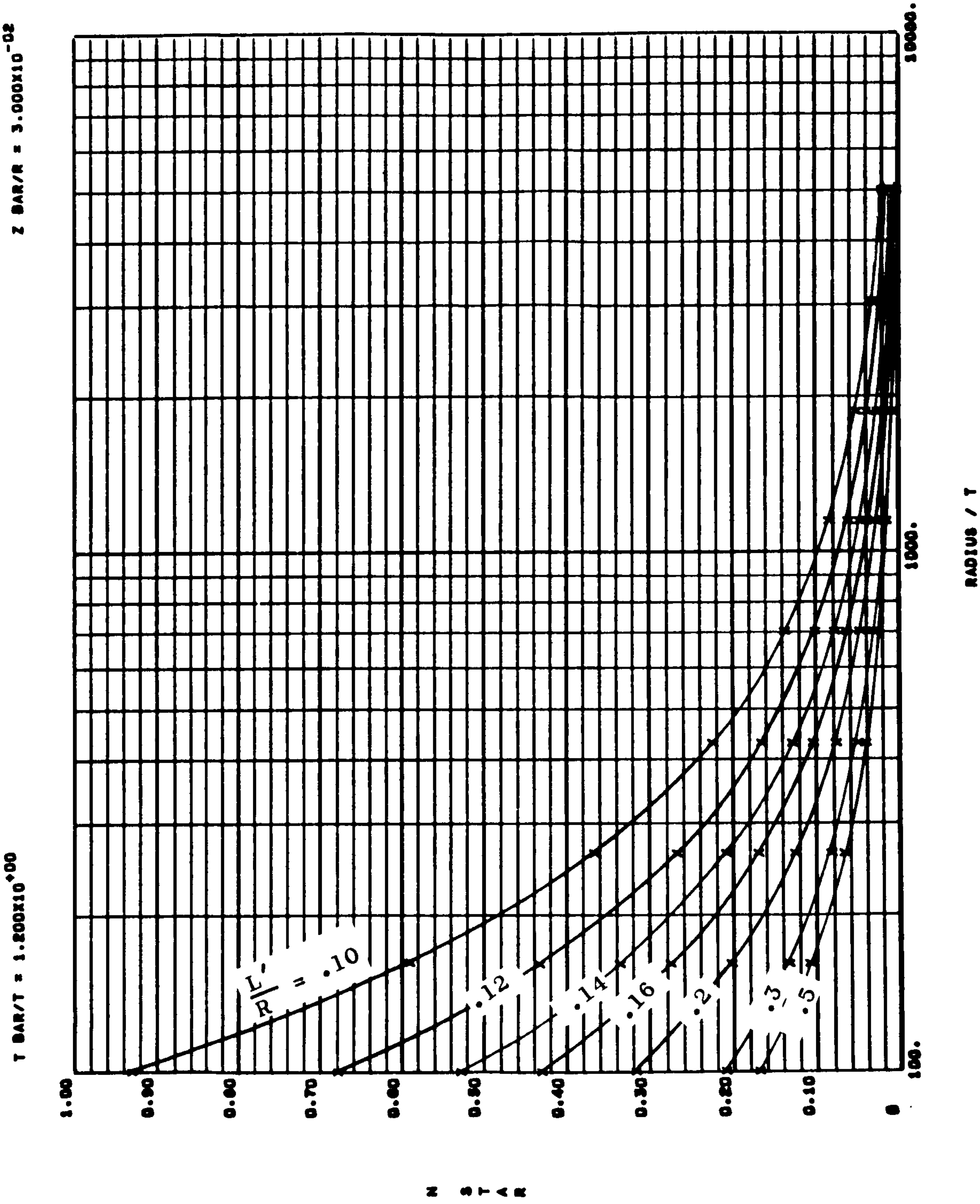
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(b)



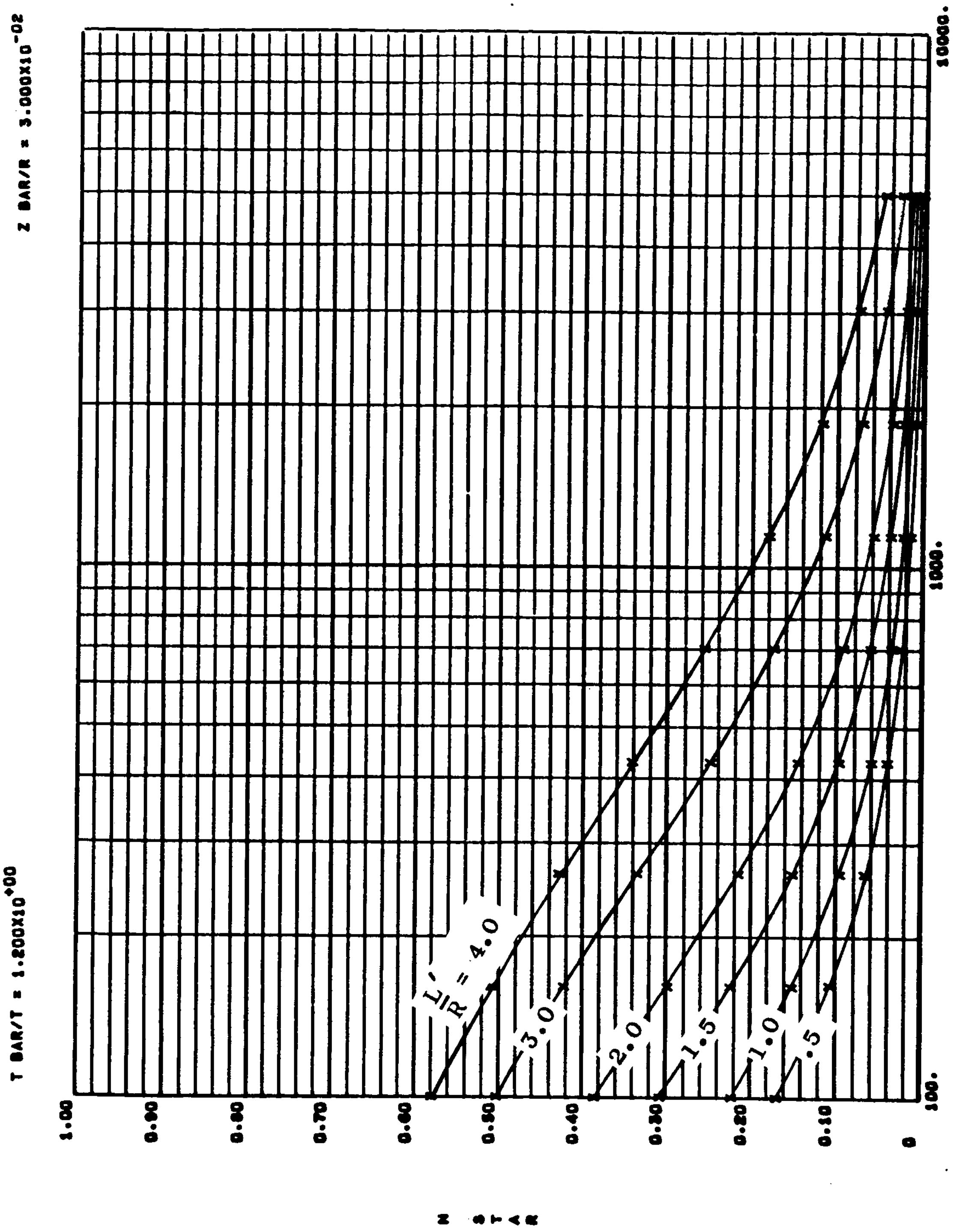
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(b)



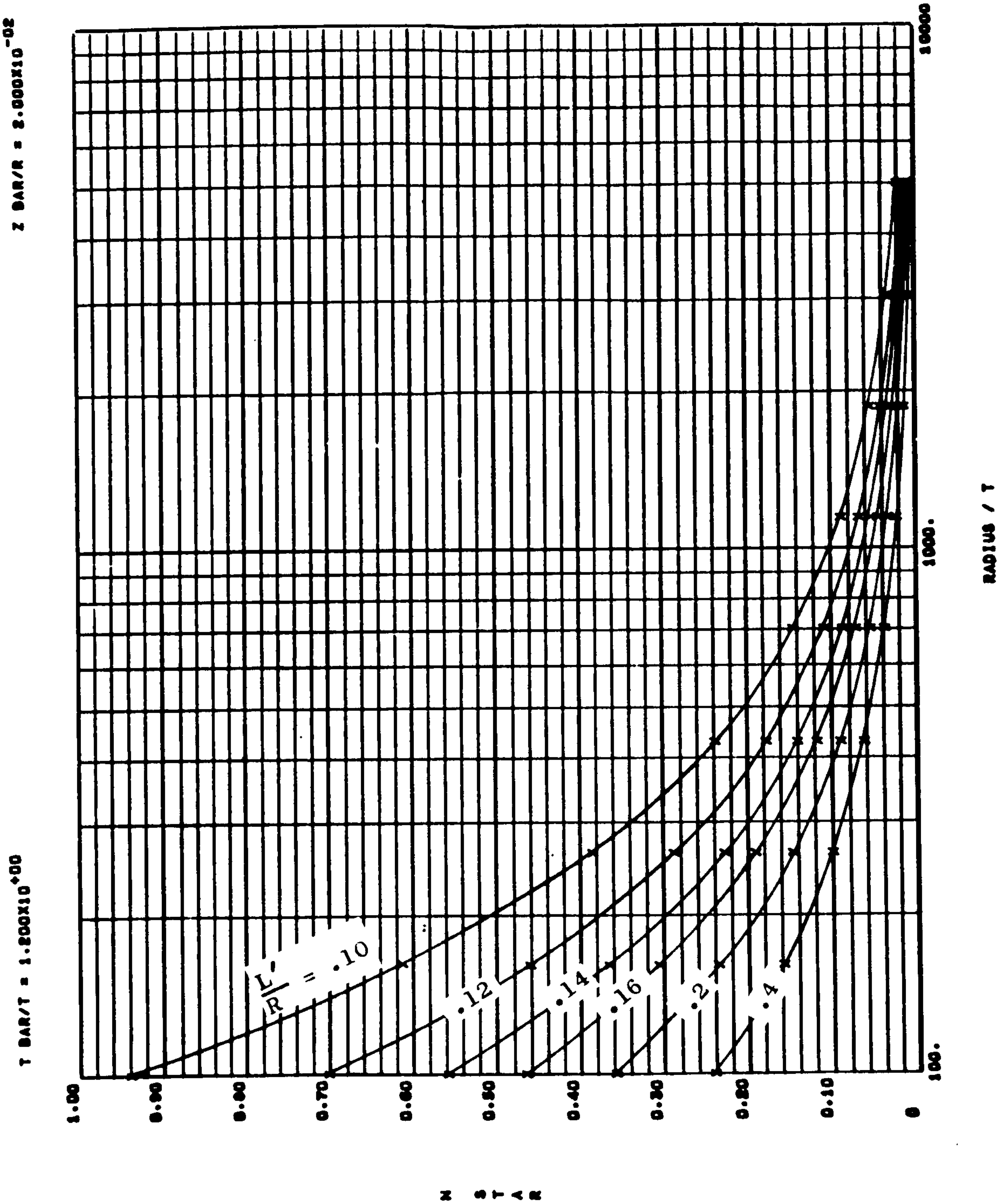
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(c)



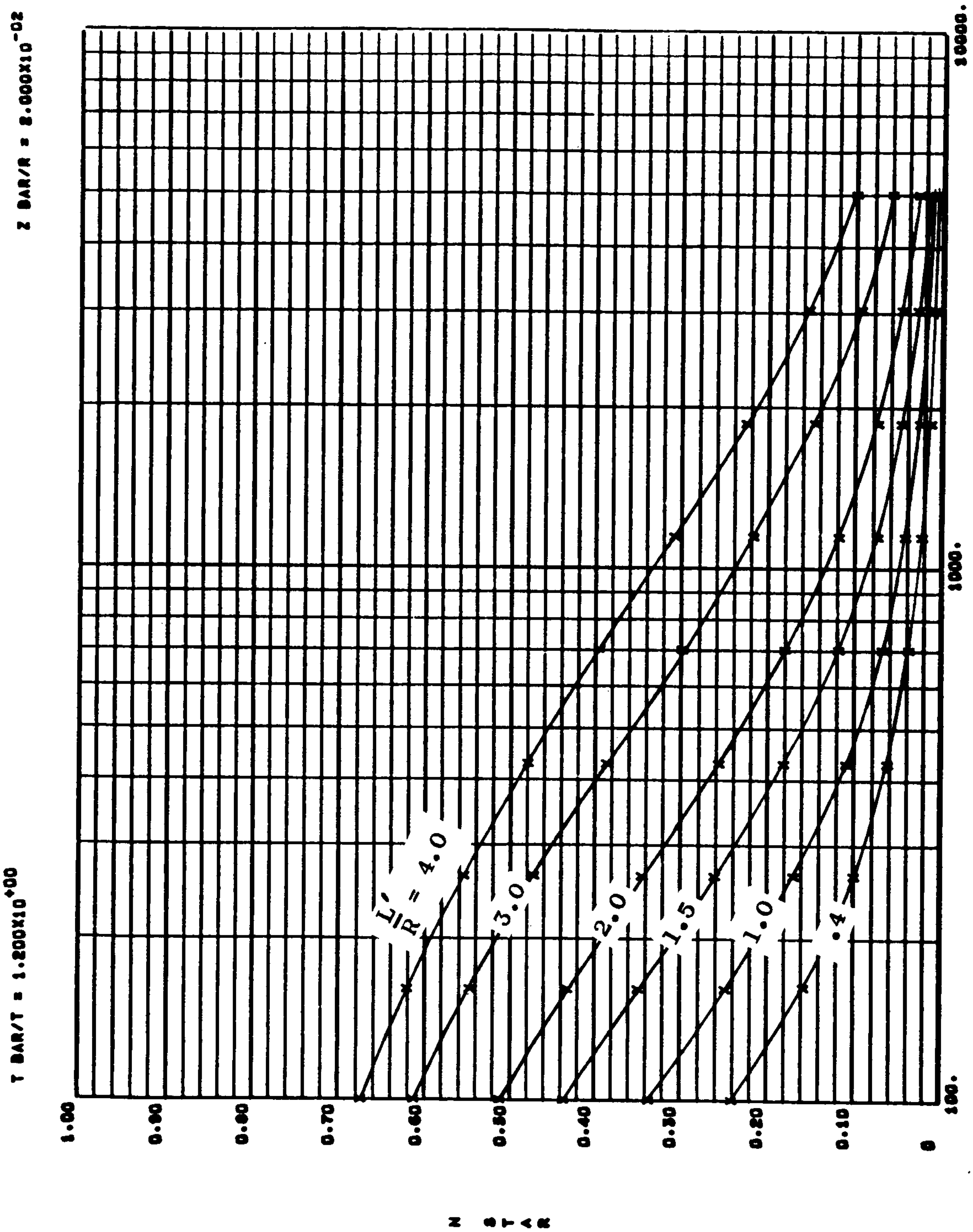
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(c)



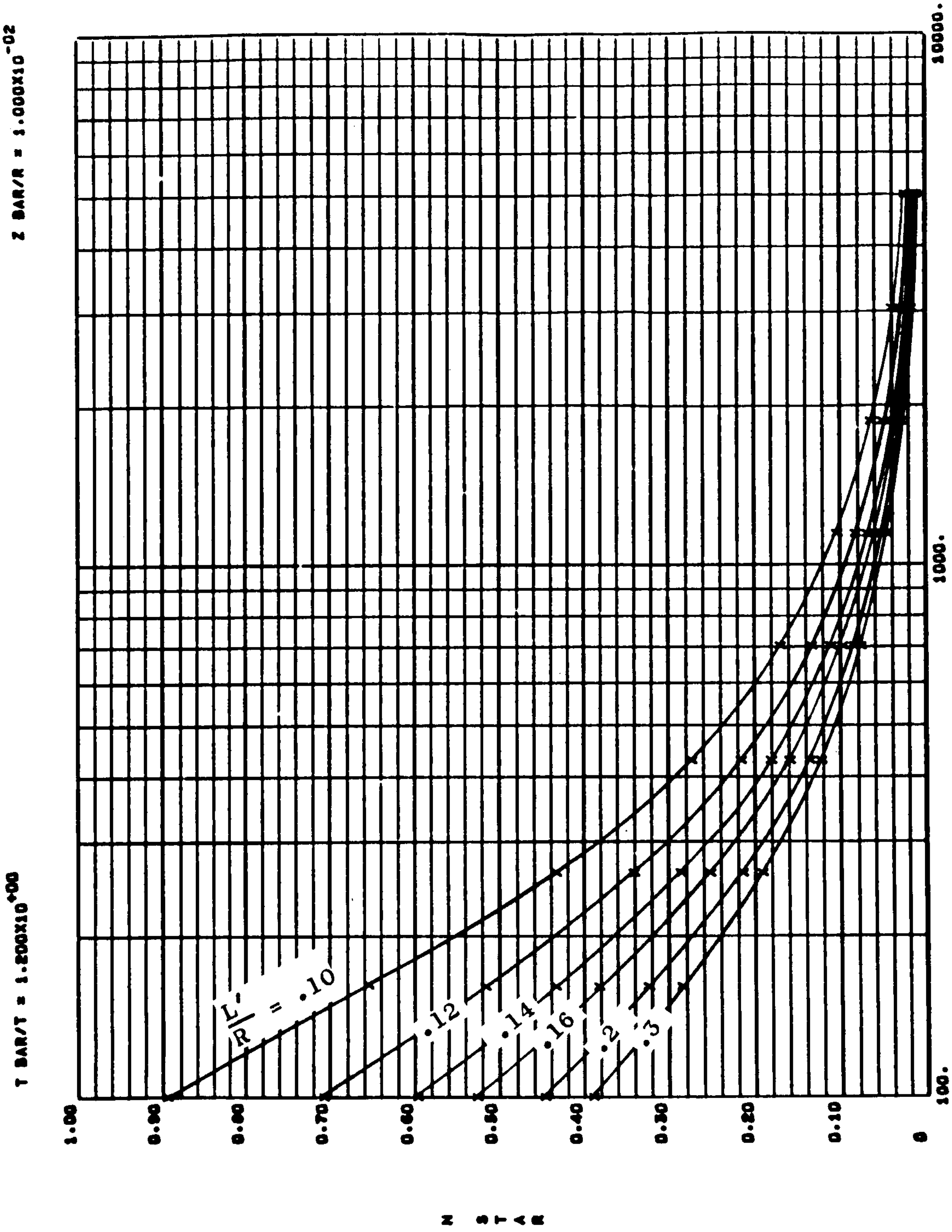
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(d)



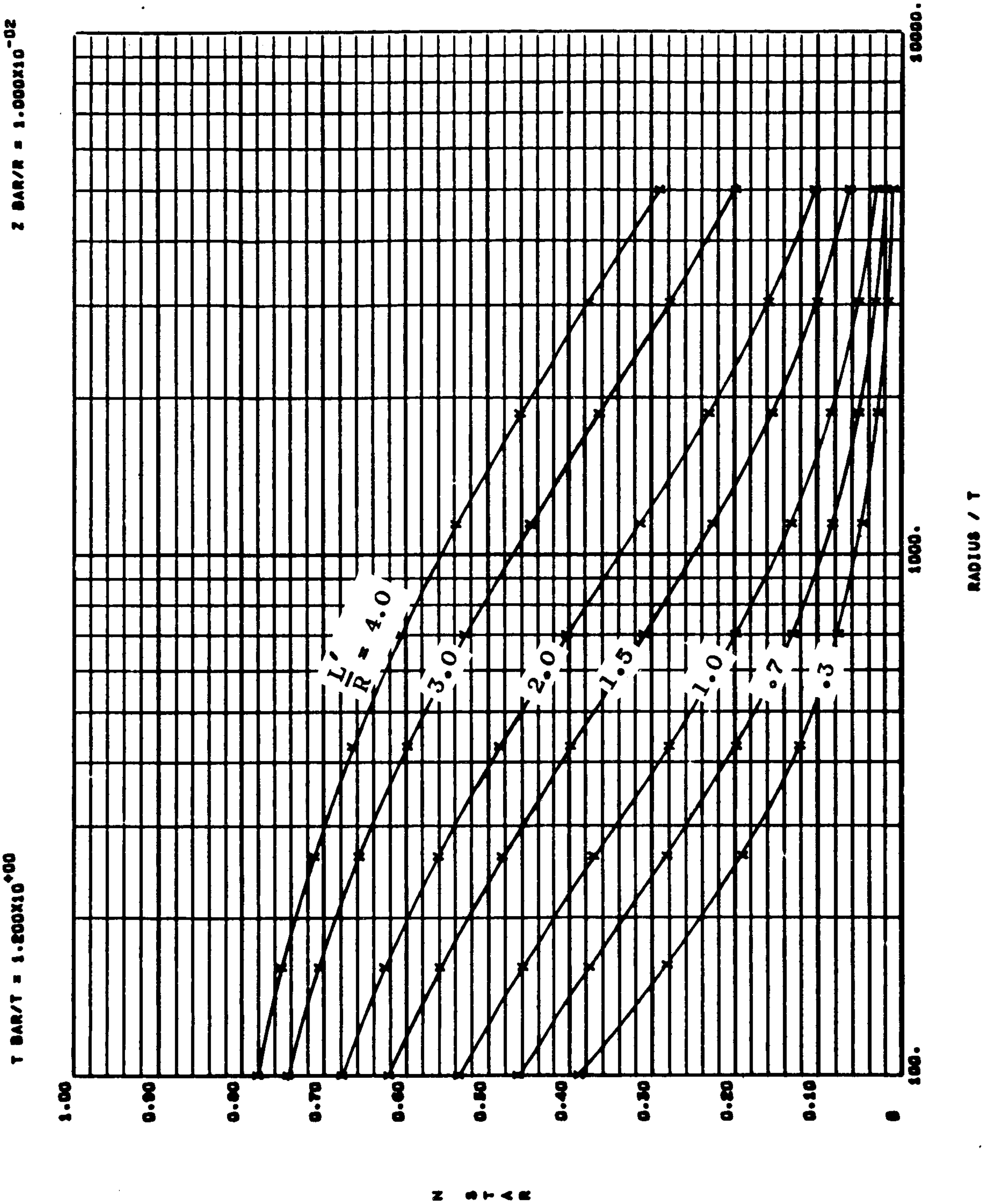
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(d)



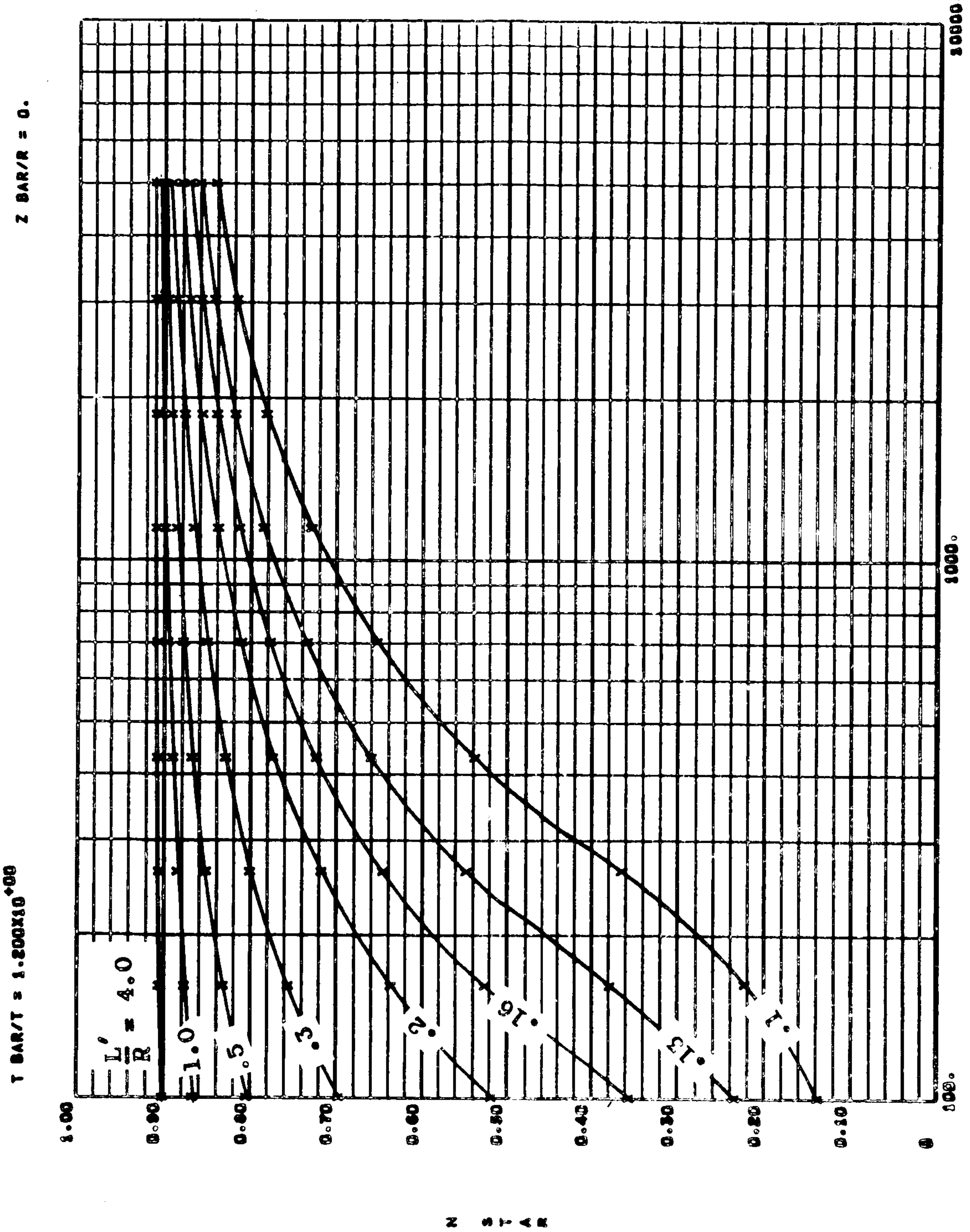
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(e)



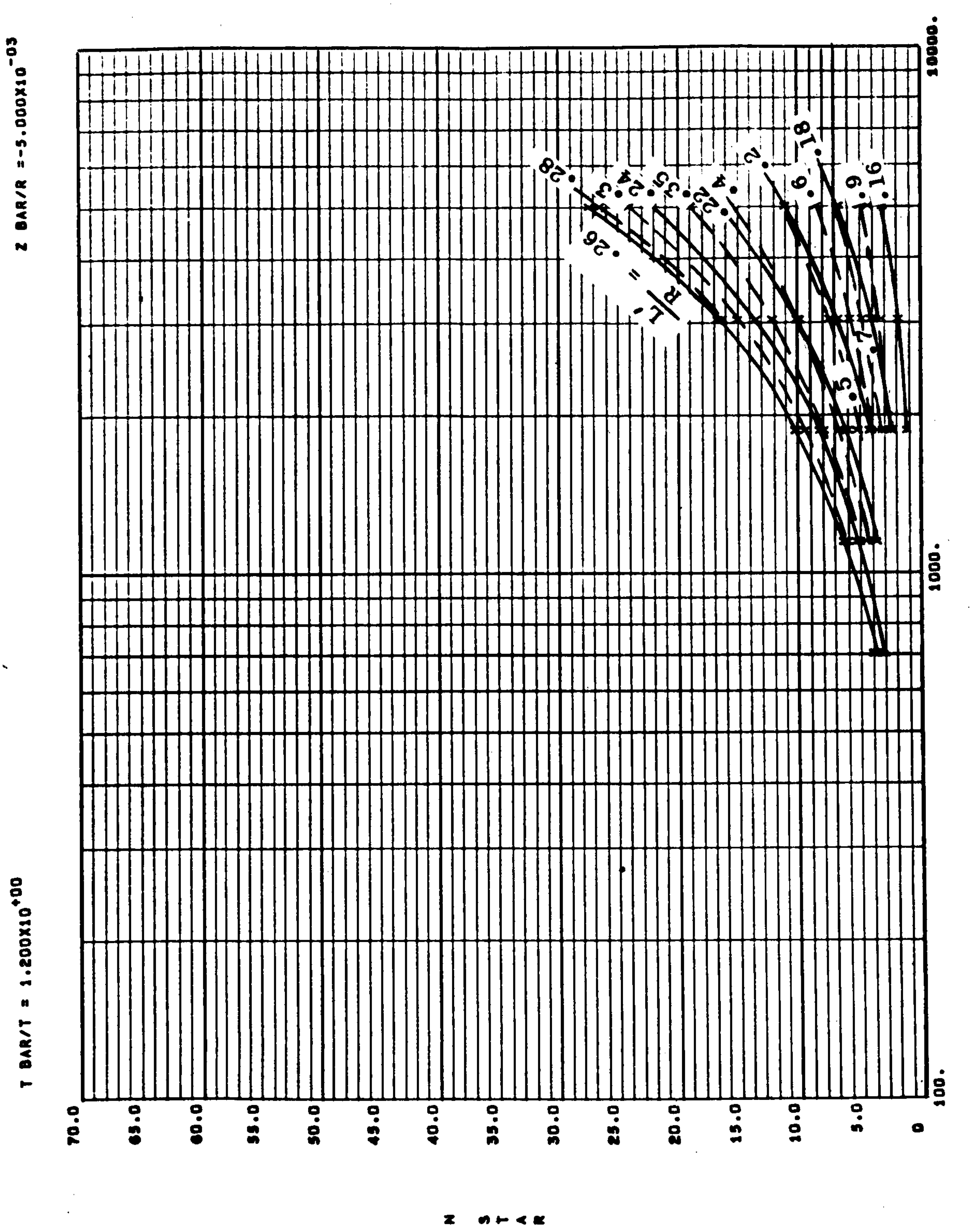
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(e)



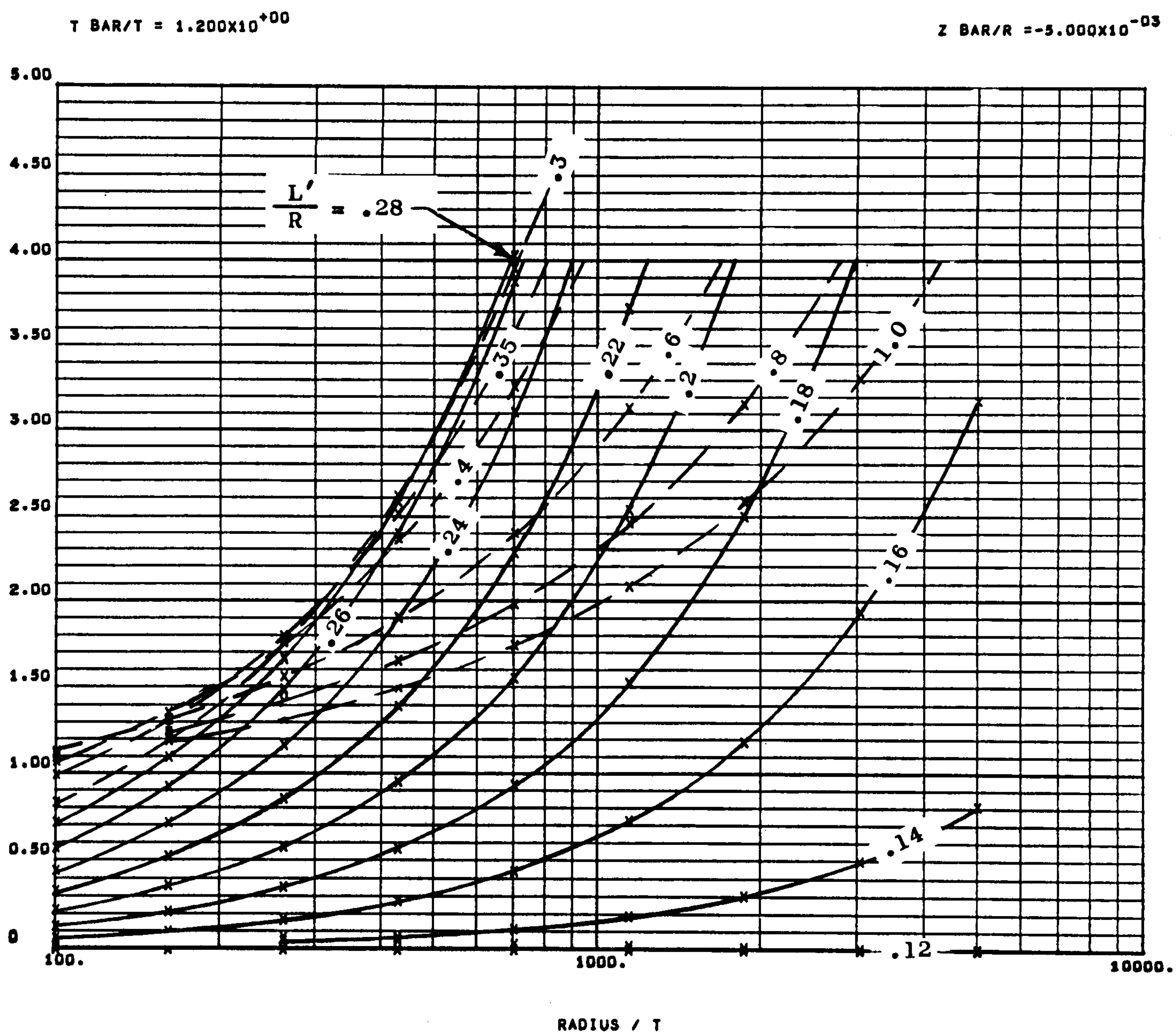
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(f)



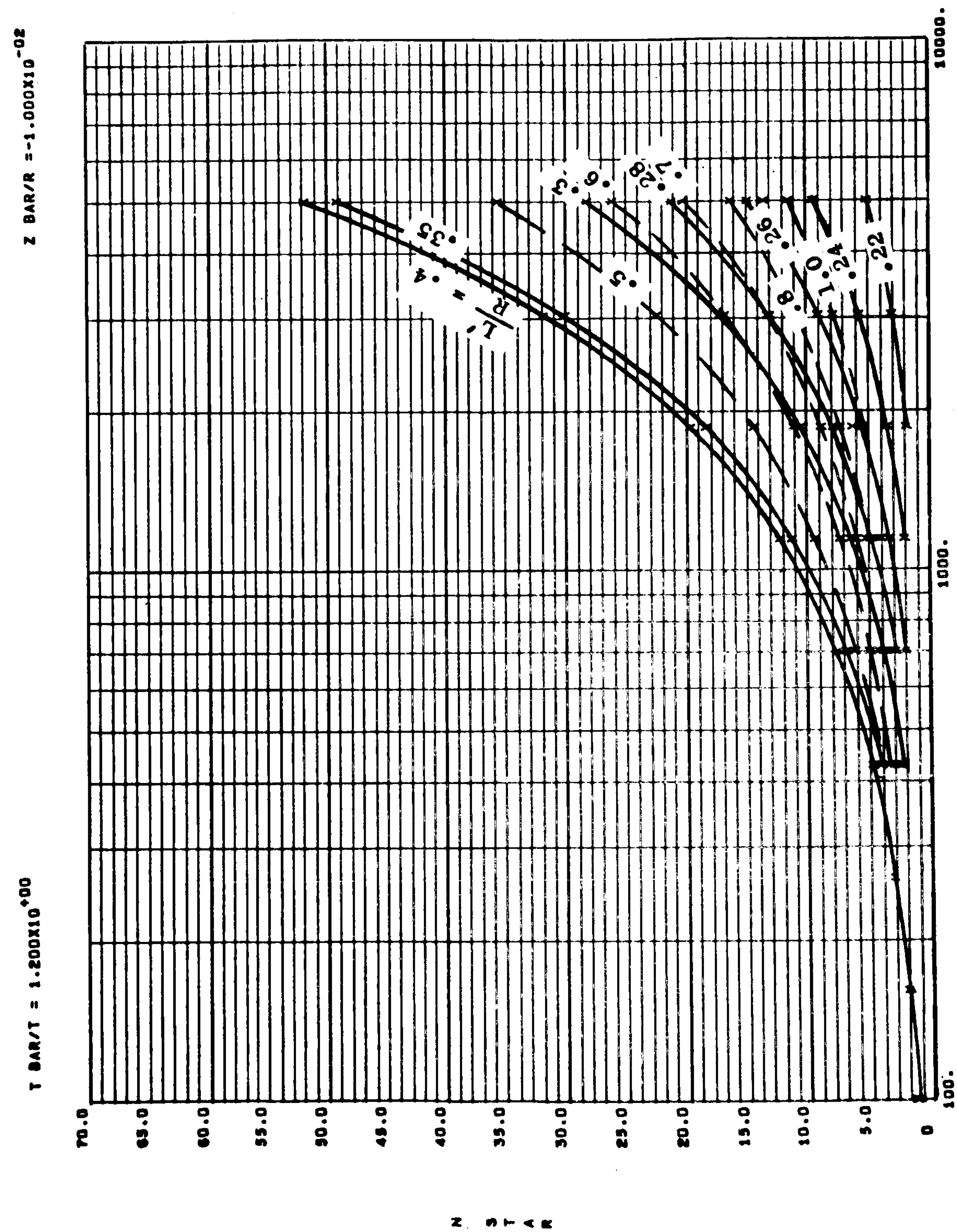
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(g)



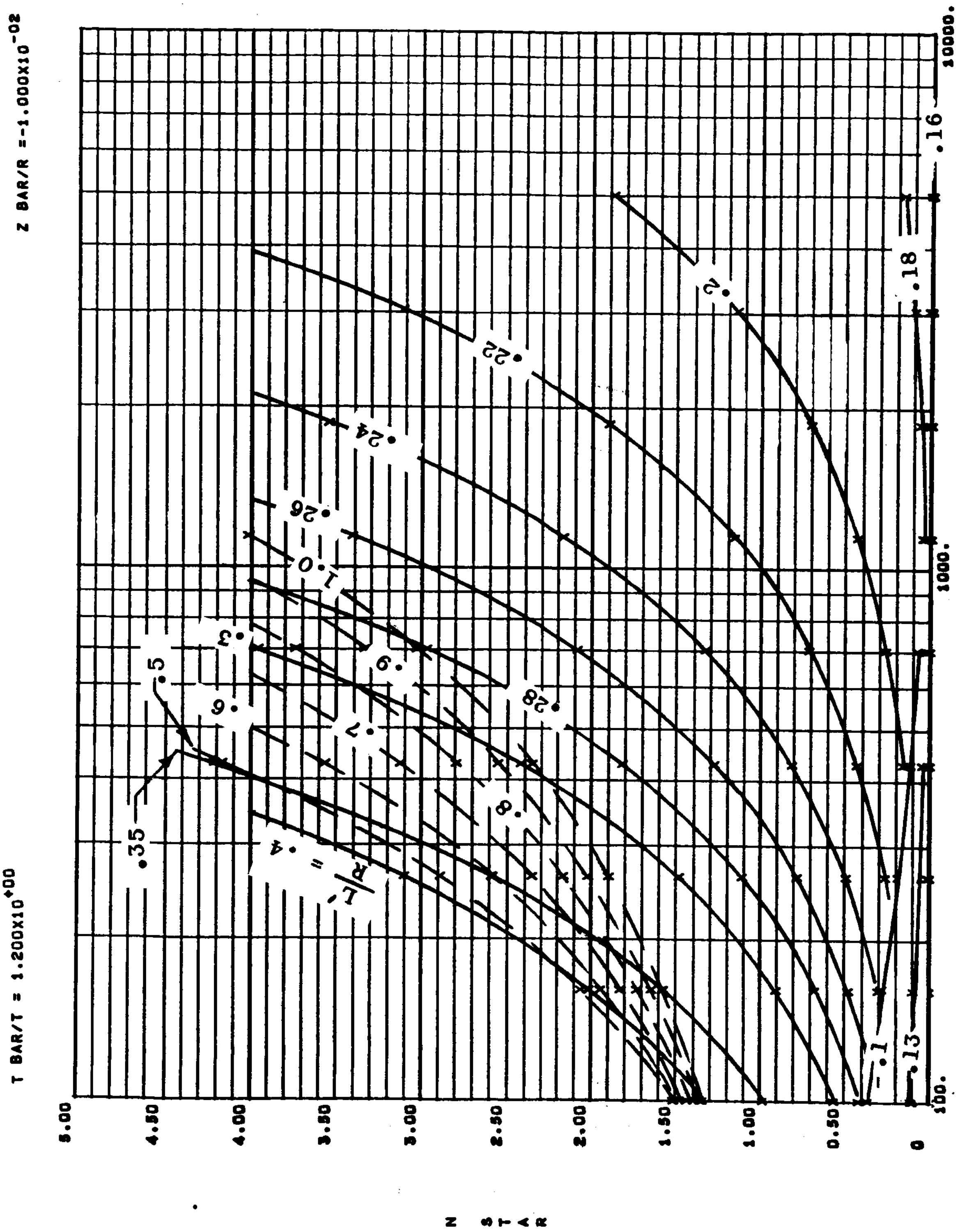
MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(g)



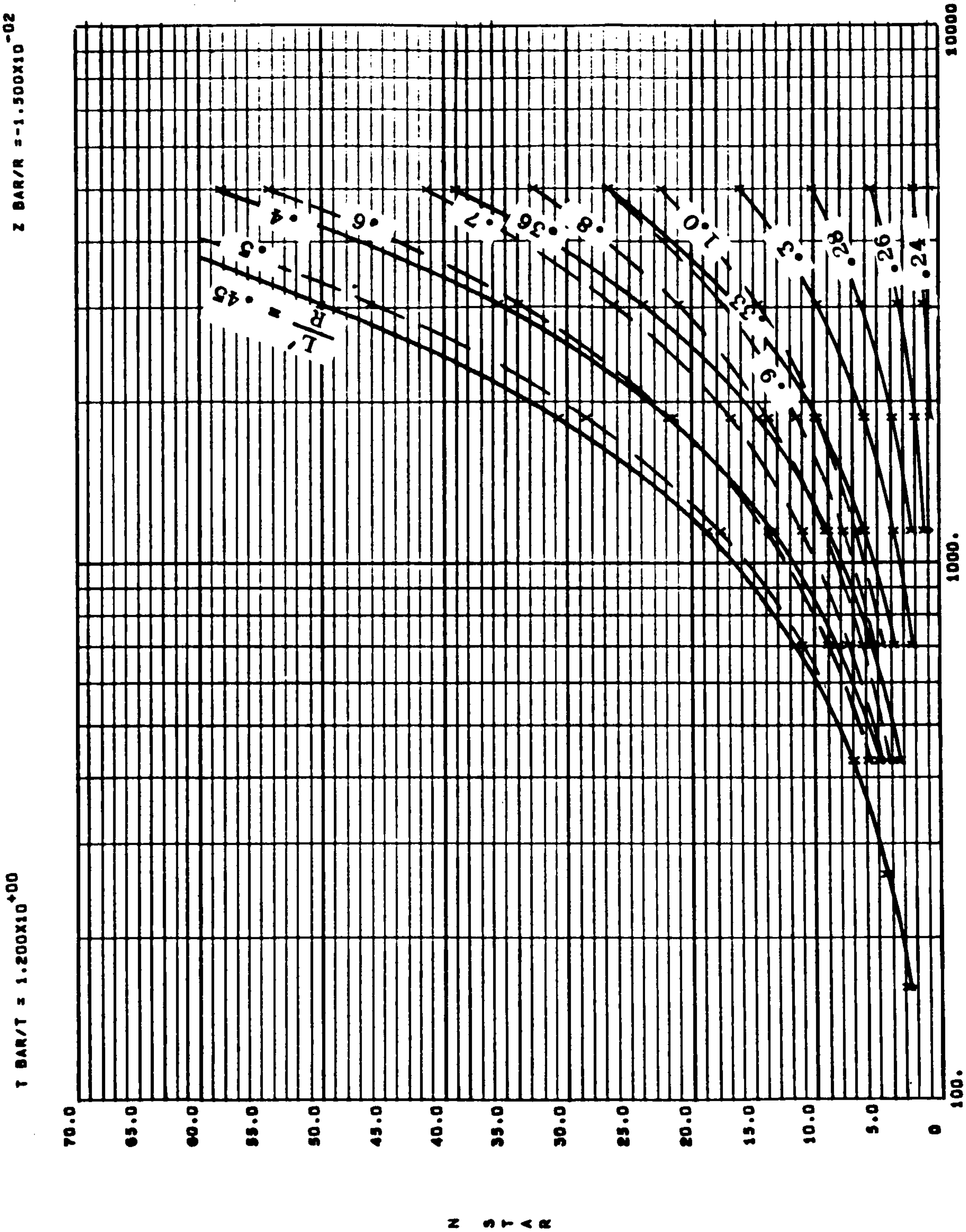
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(h)



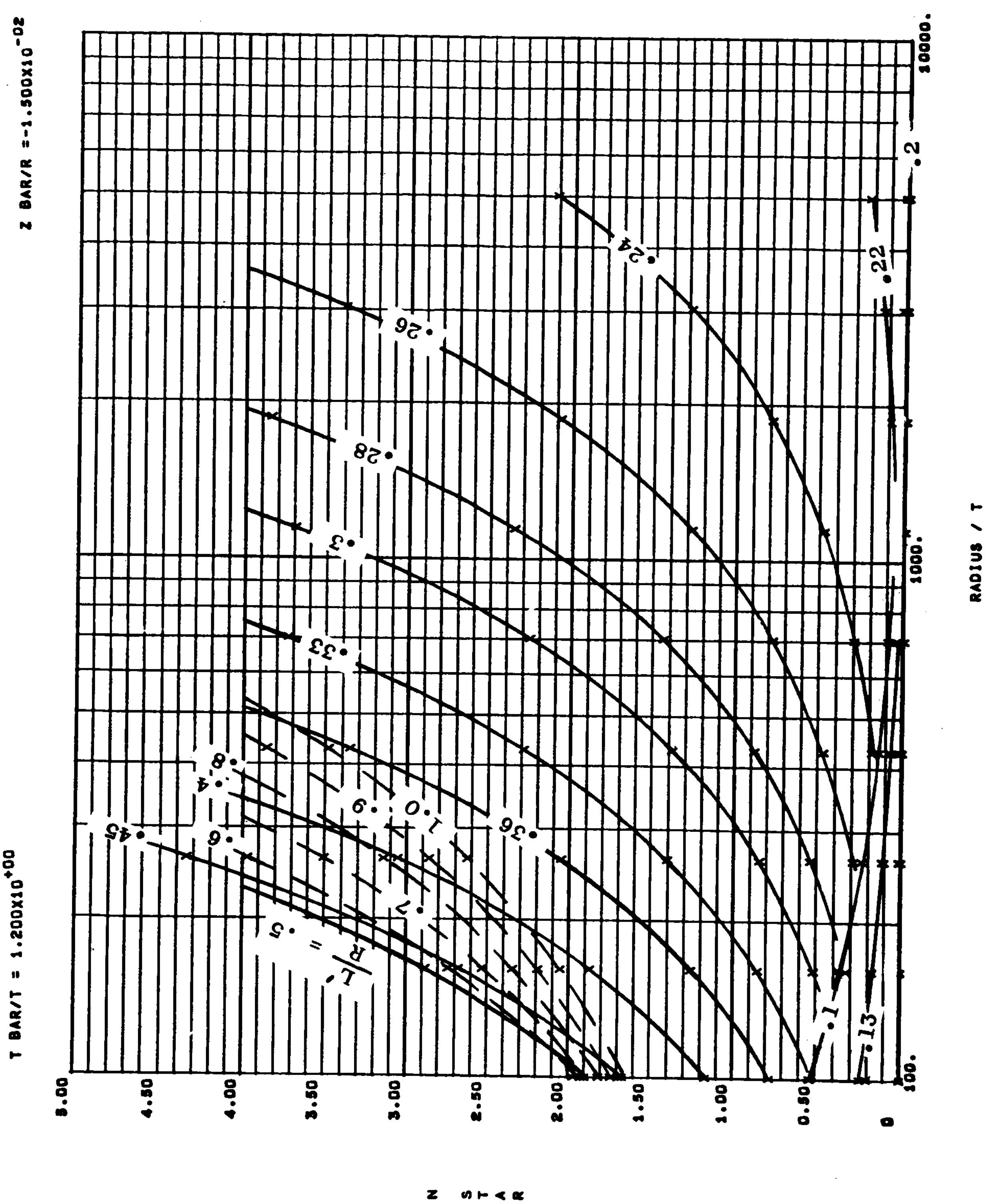
MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(h)



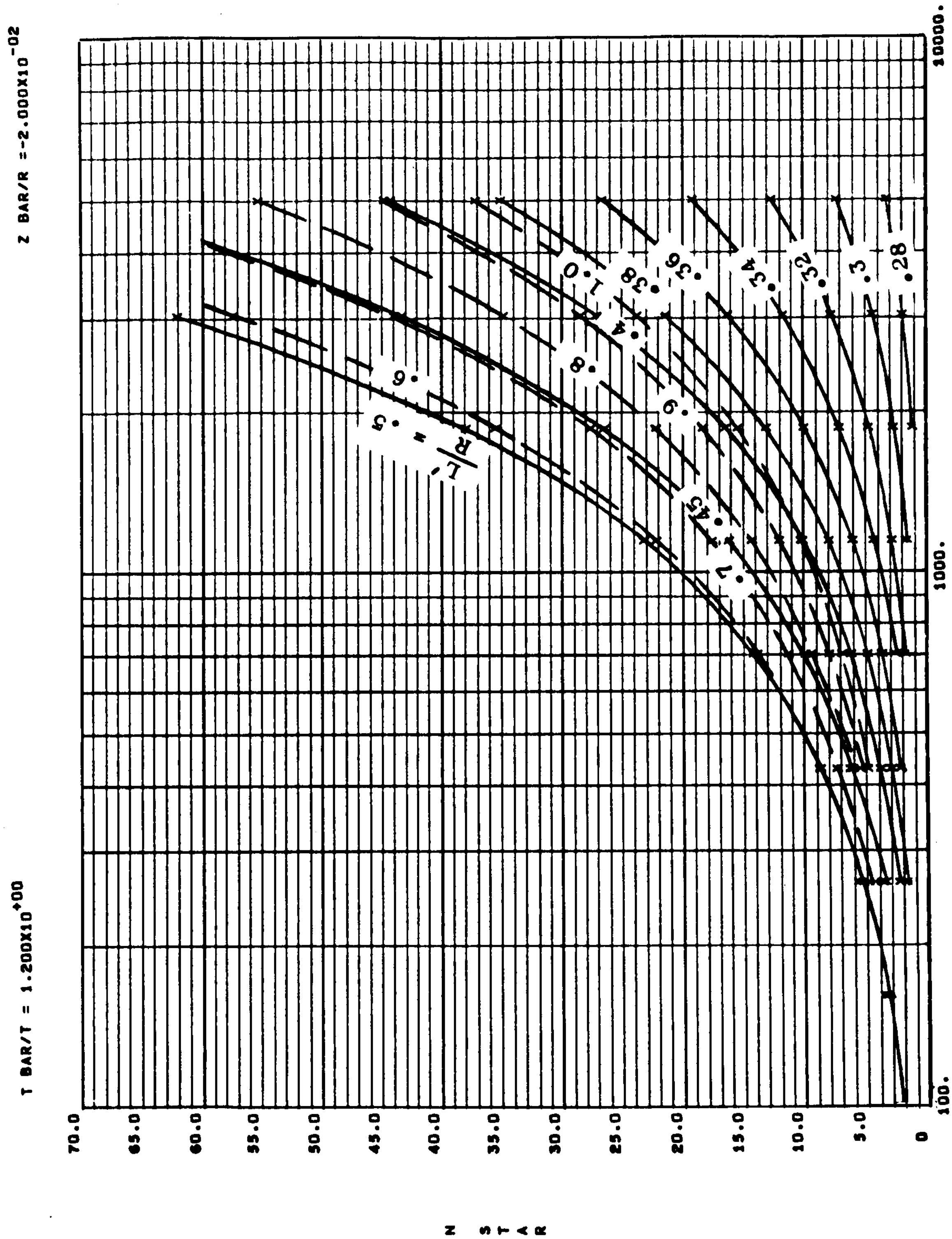
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(i)



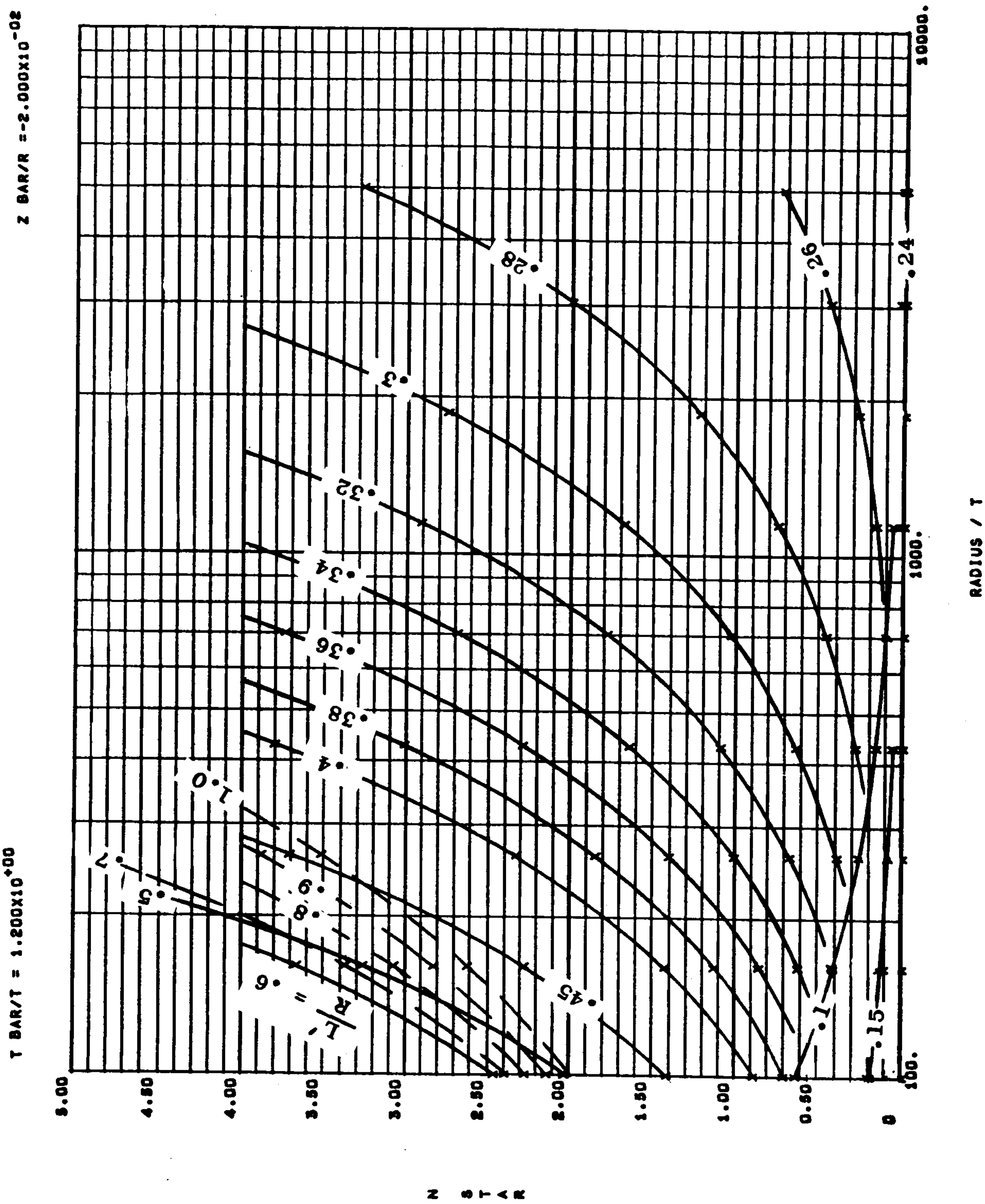
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(i)



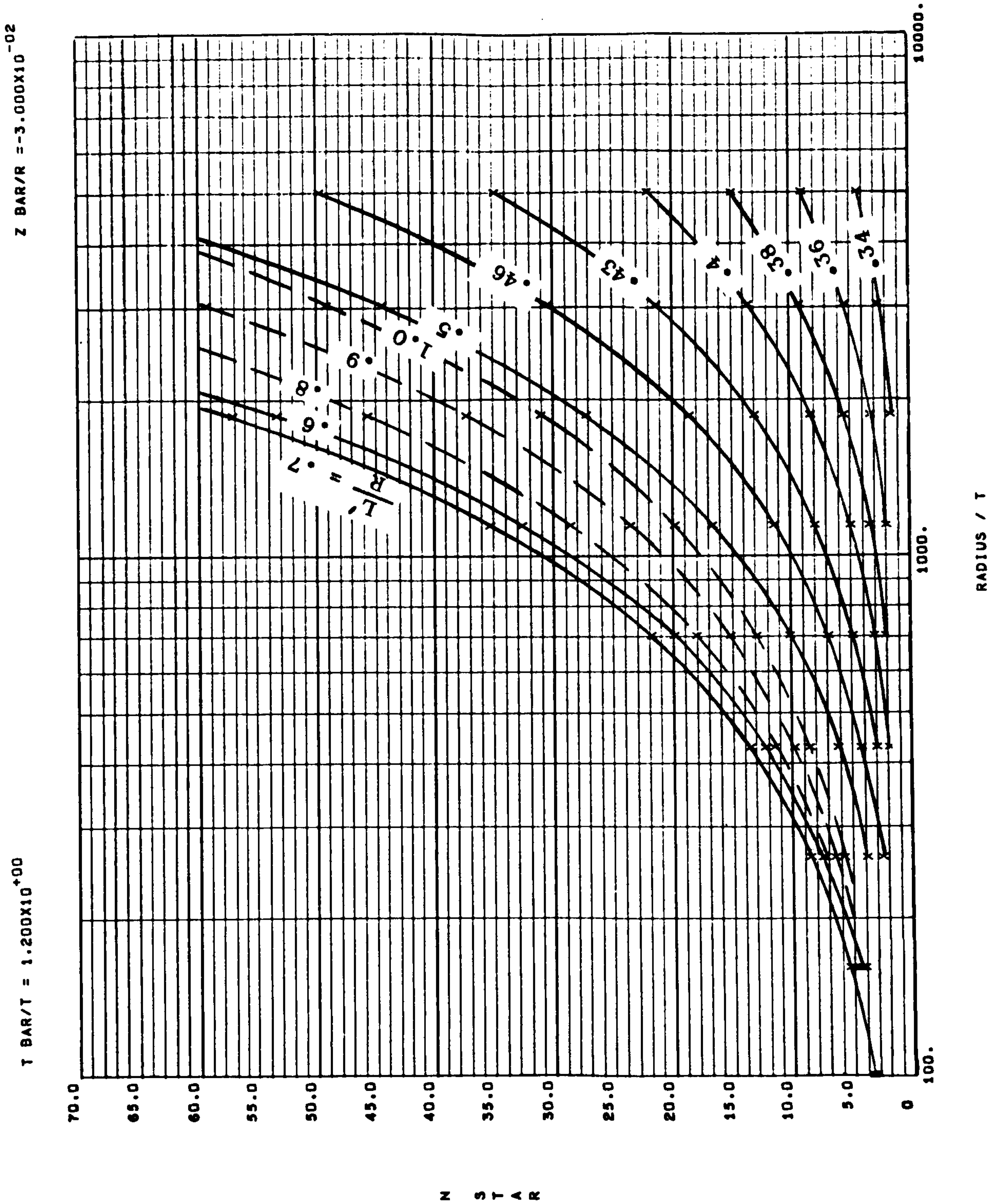
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(j)



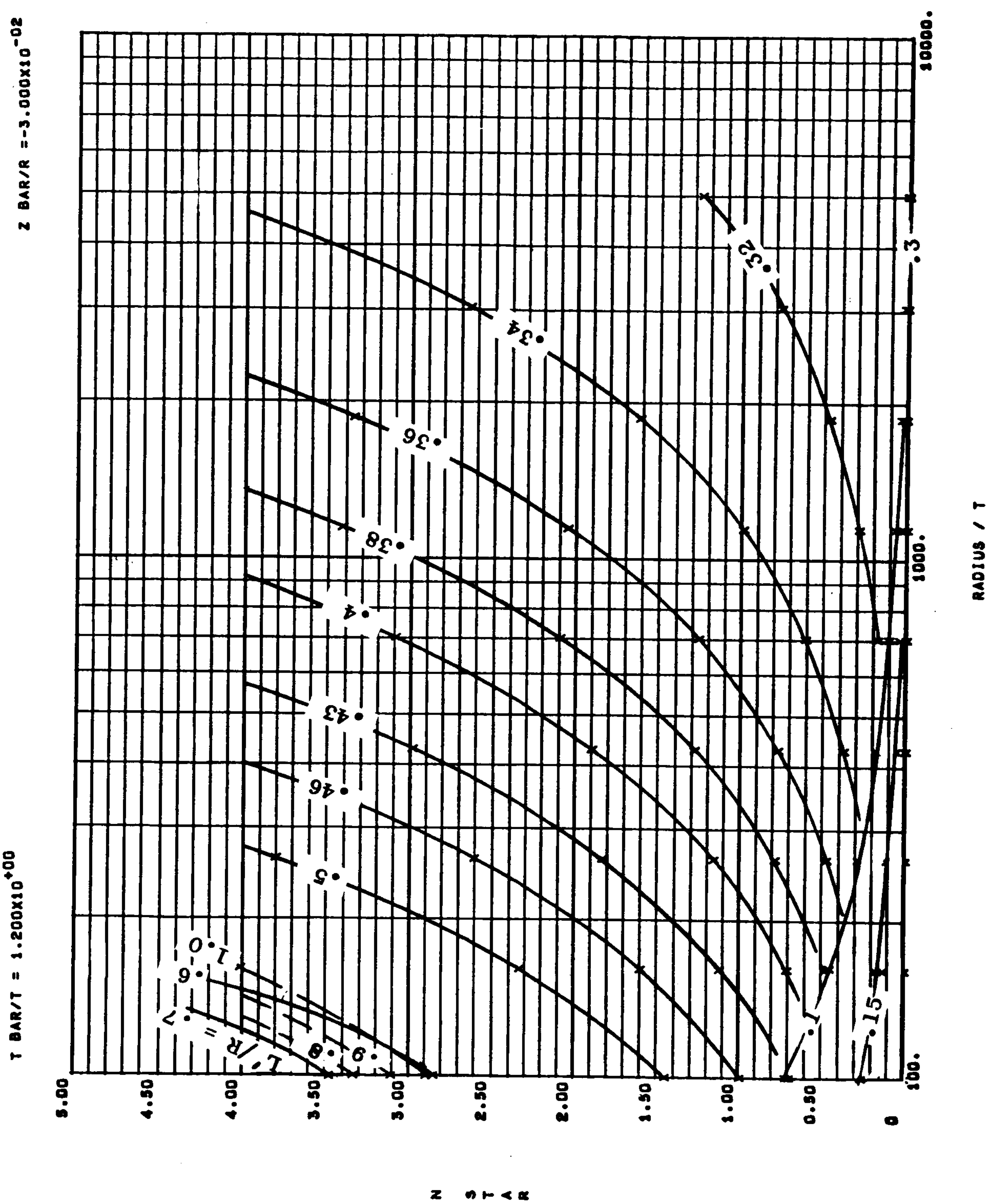
MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(j)



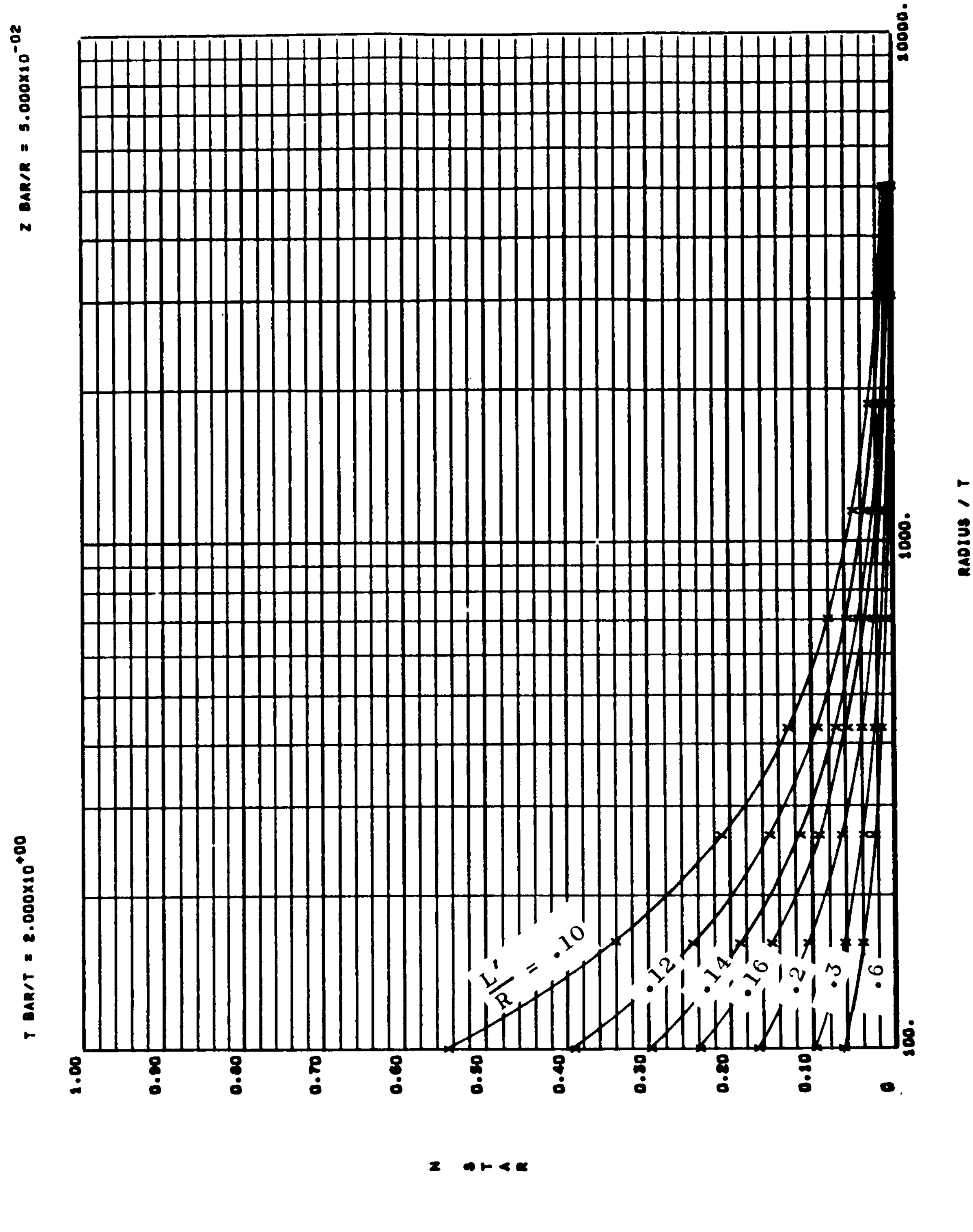
MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(k)



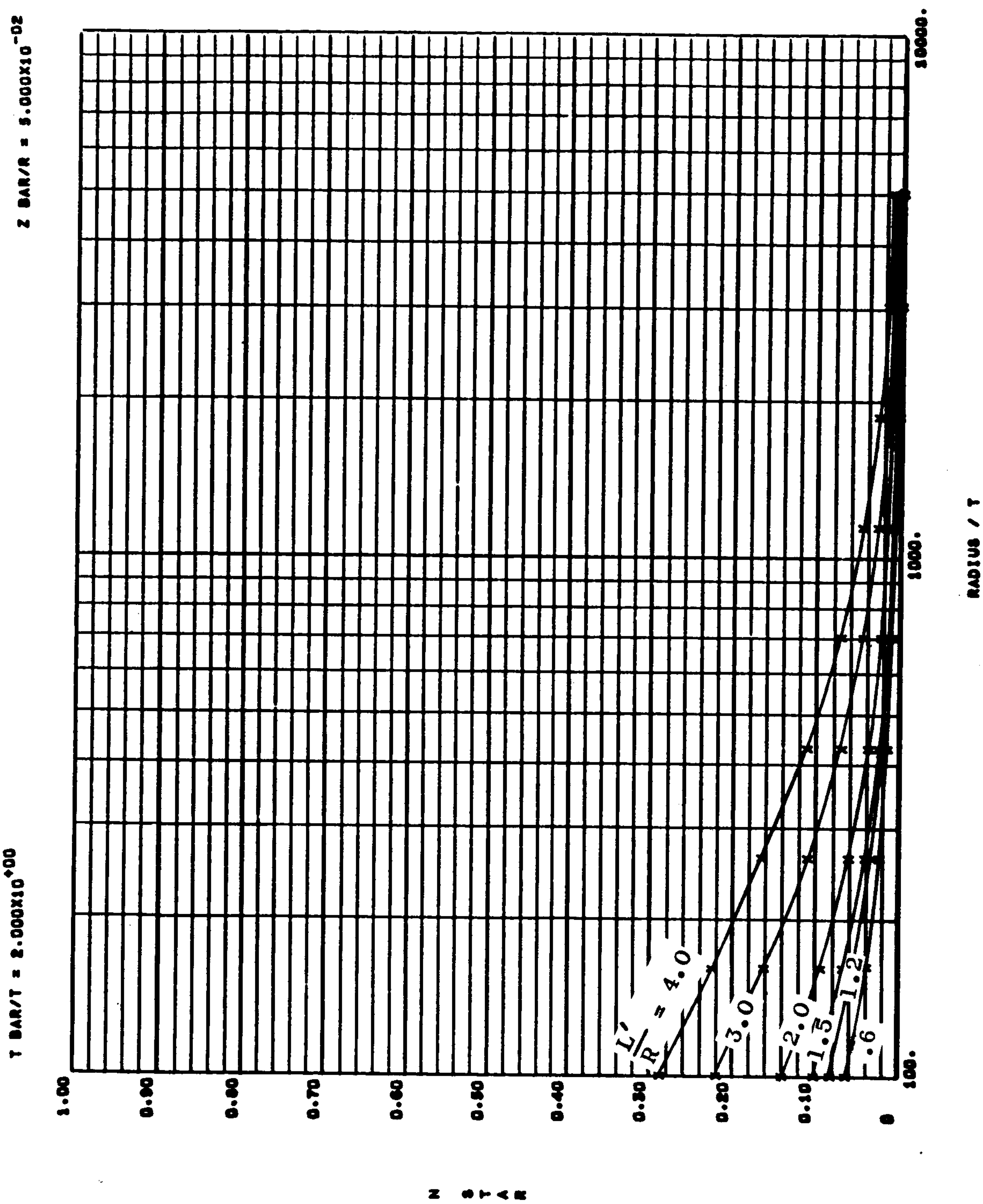
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(k)



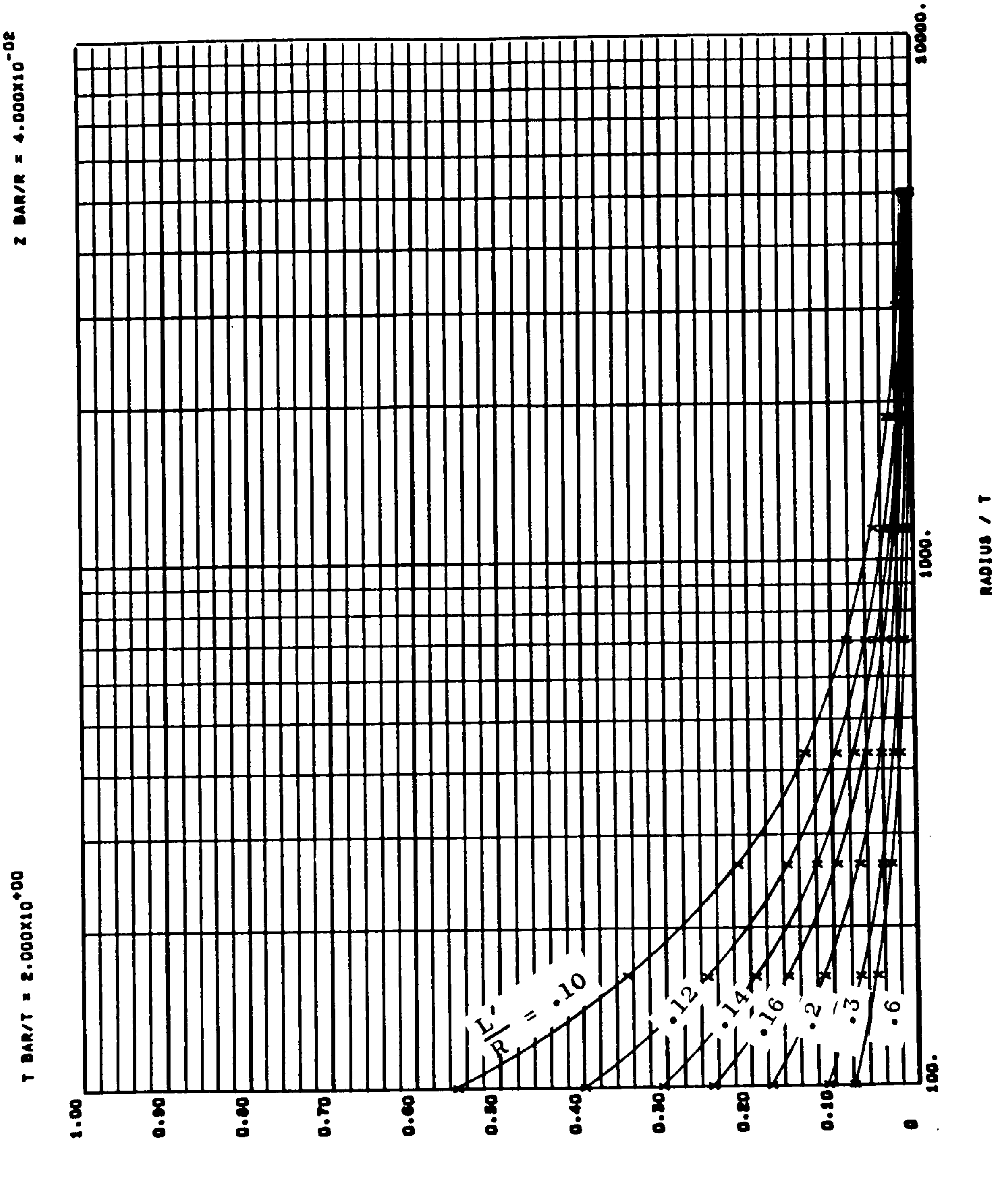
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(1)



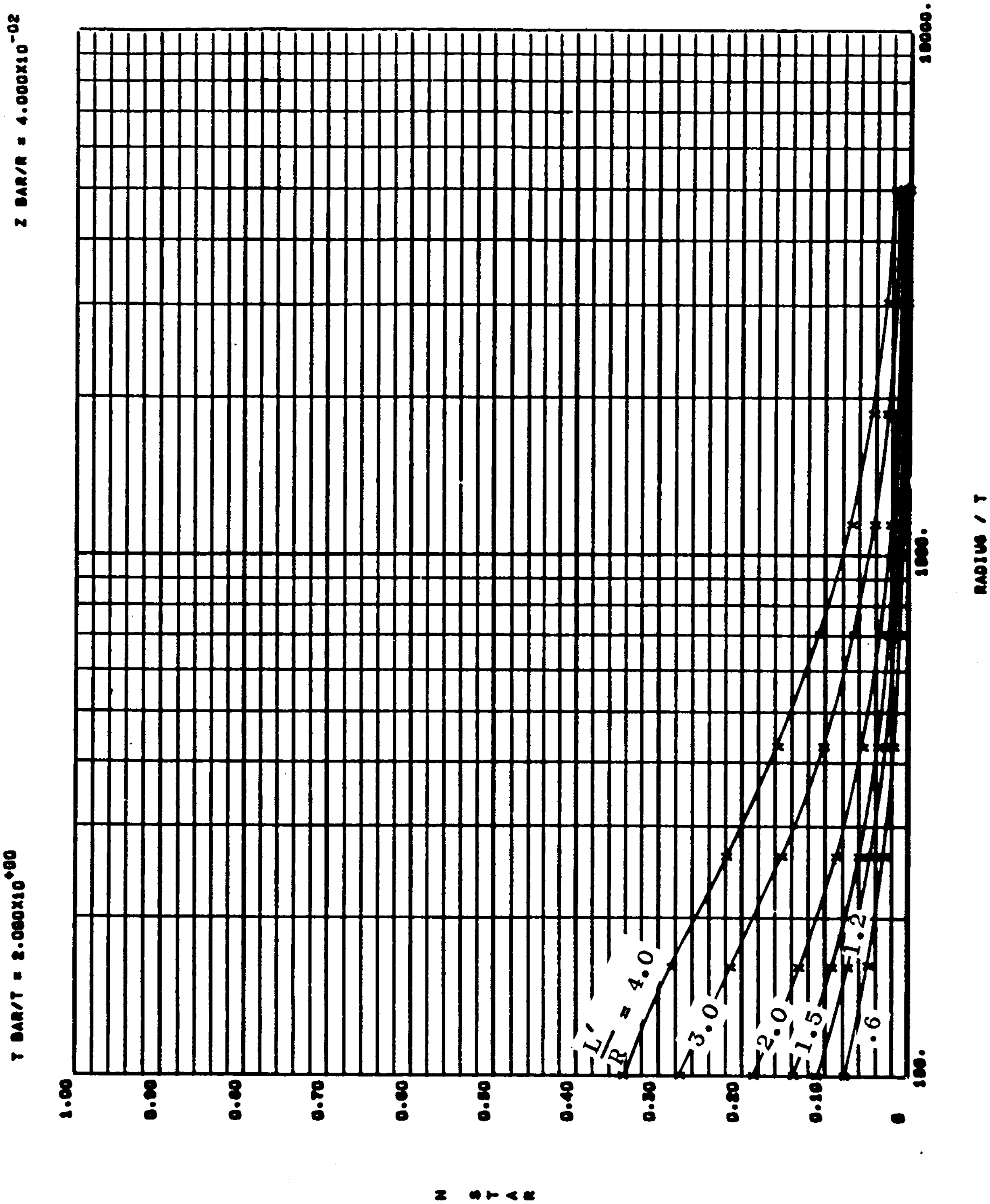
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(1)



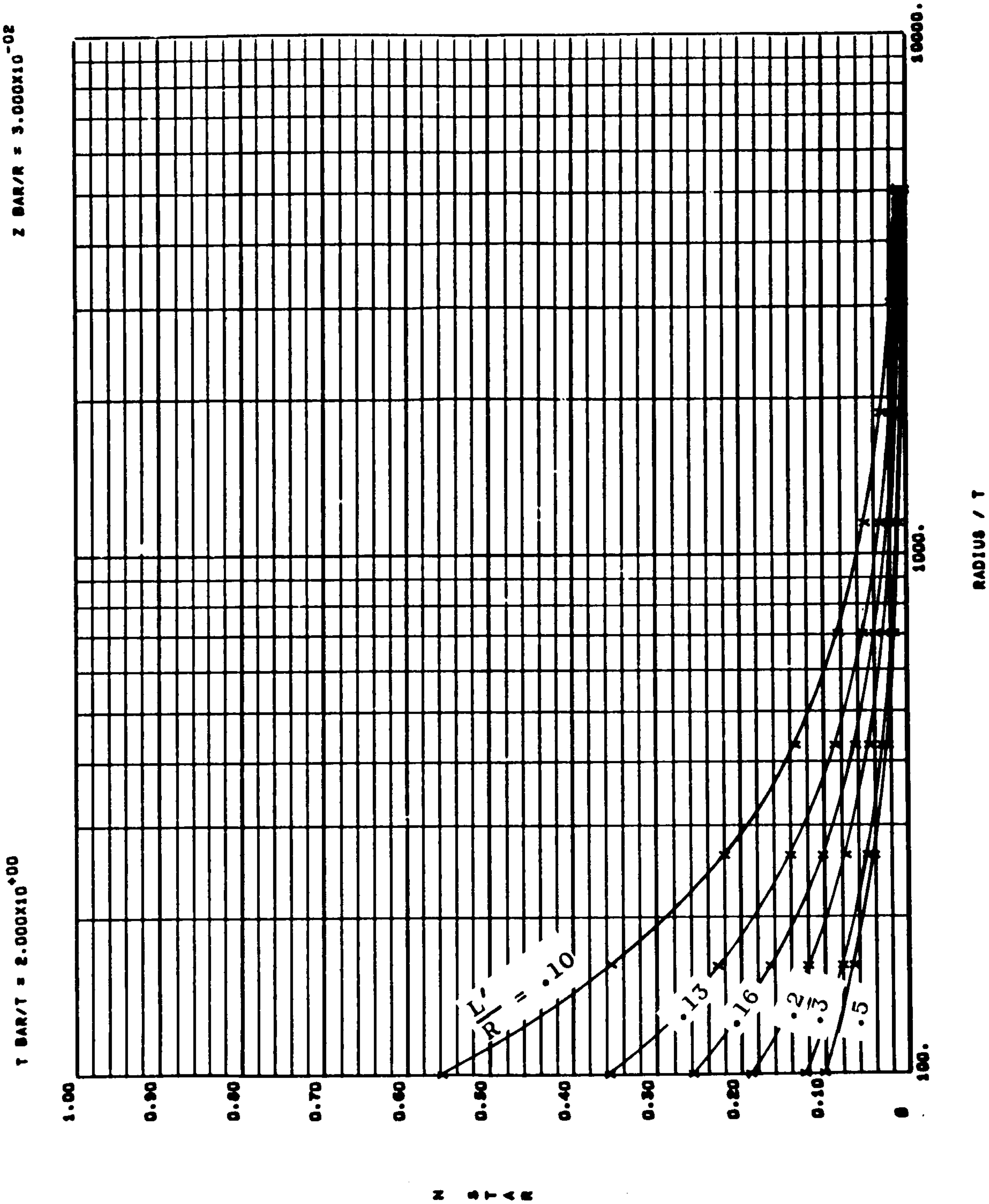
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(m)



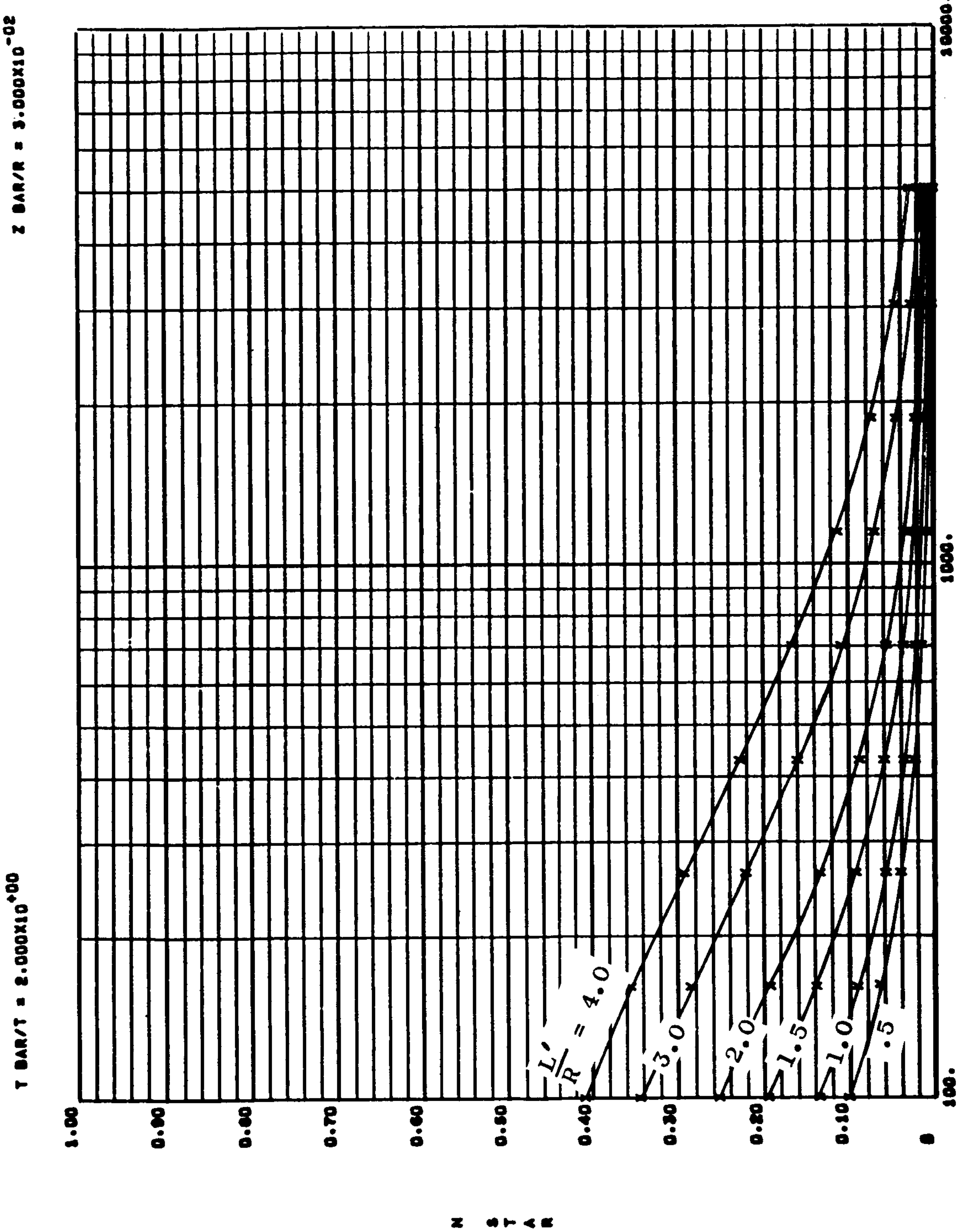
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(m)



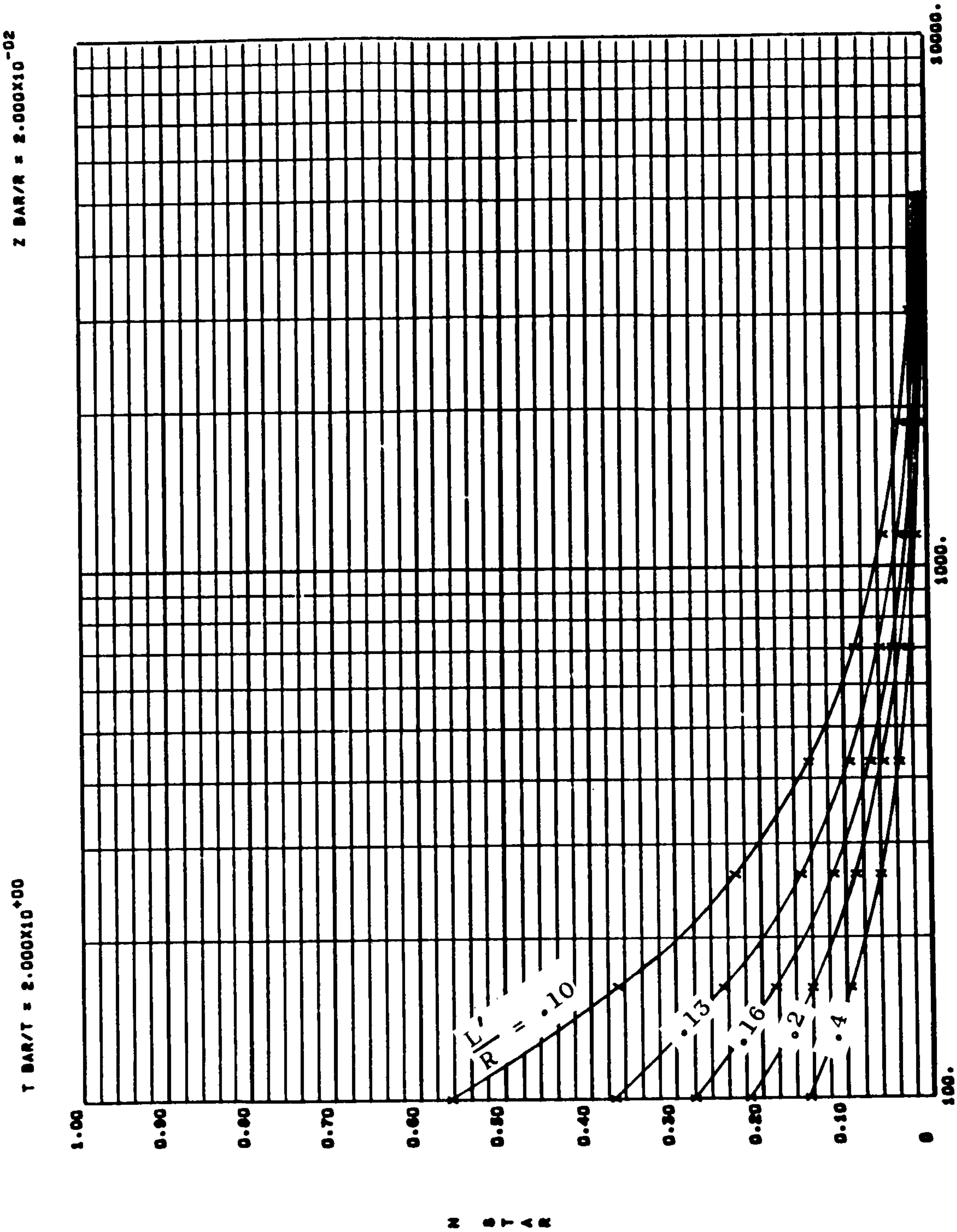
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(n)



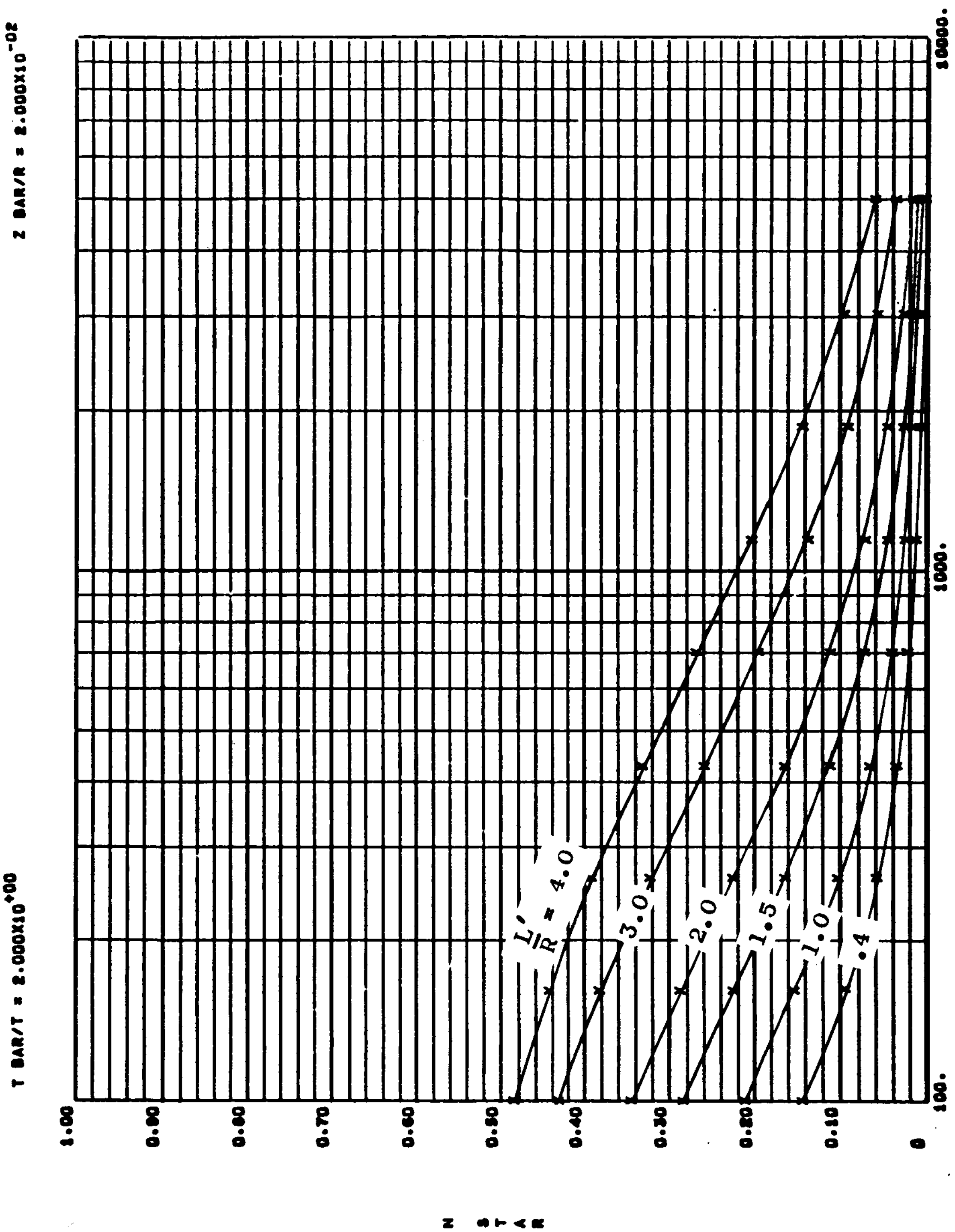
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(n)



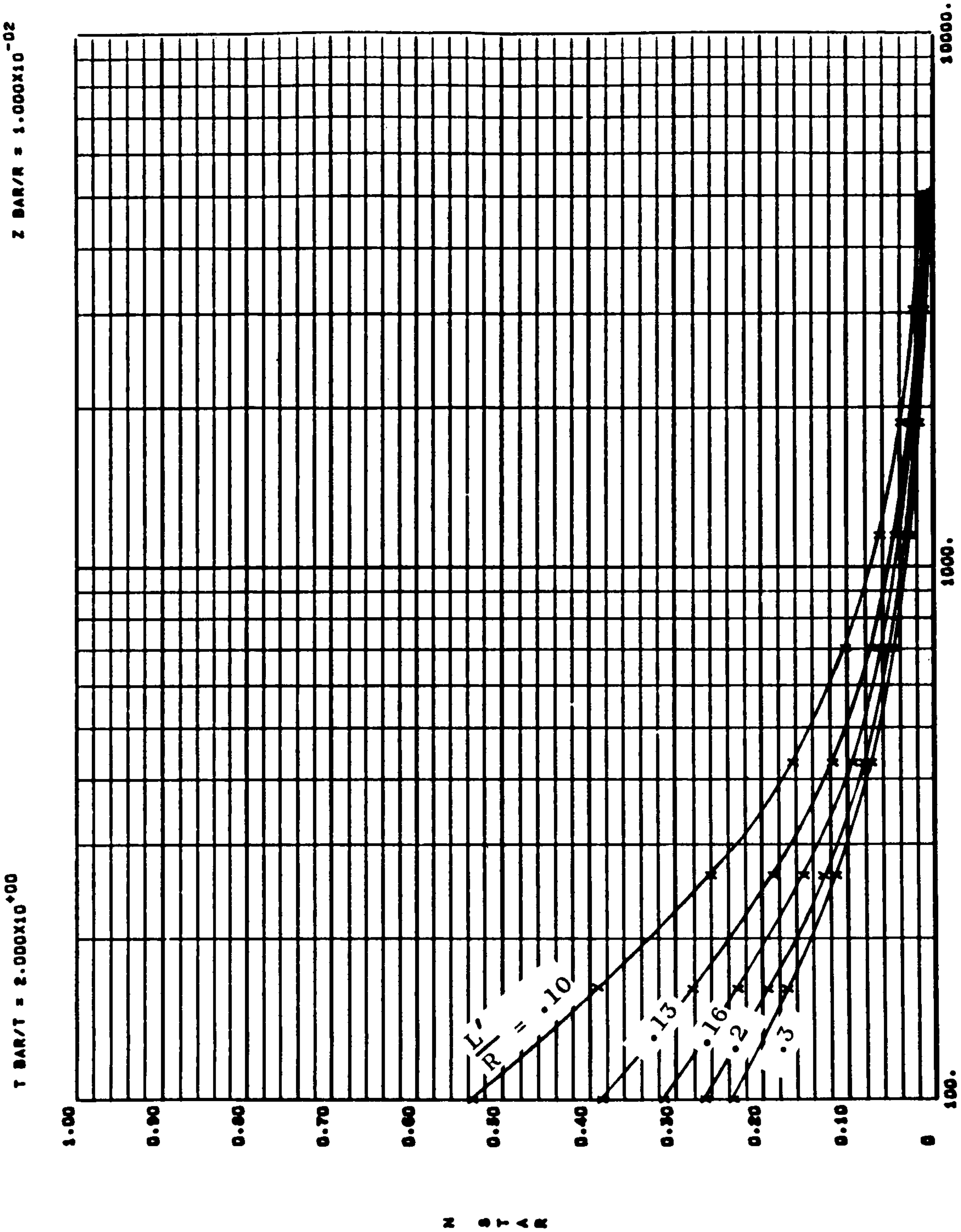
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(o)



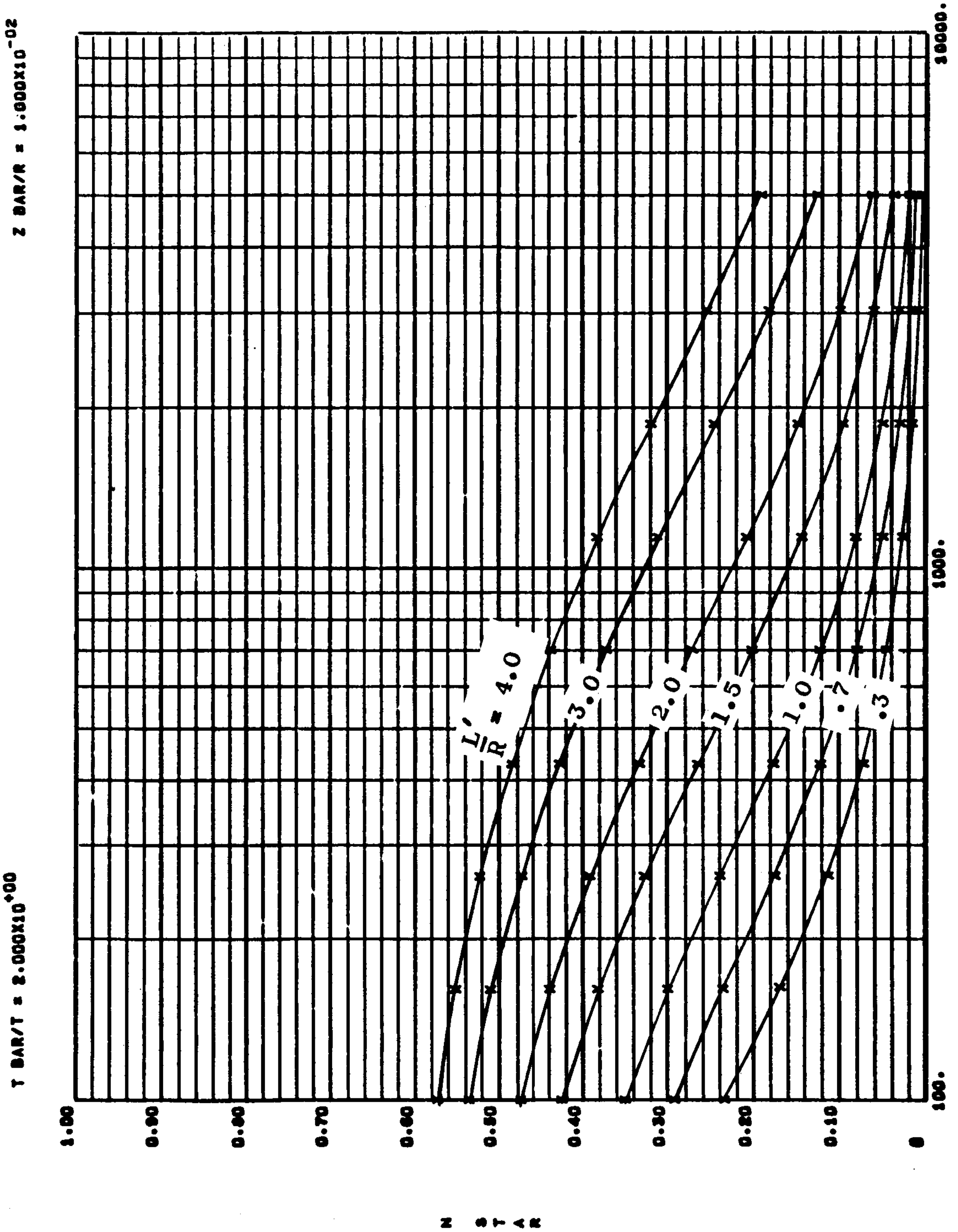
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(o)



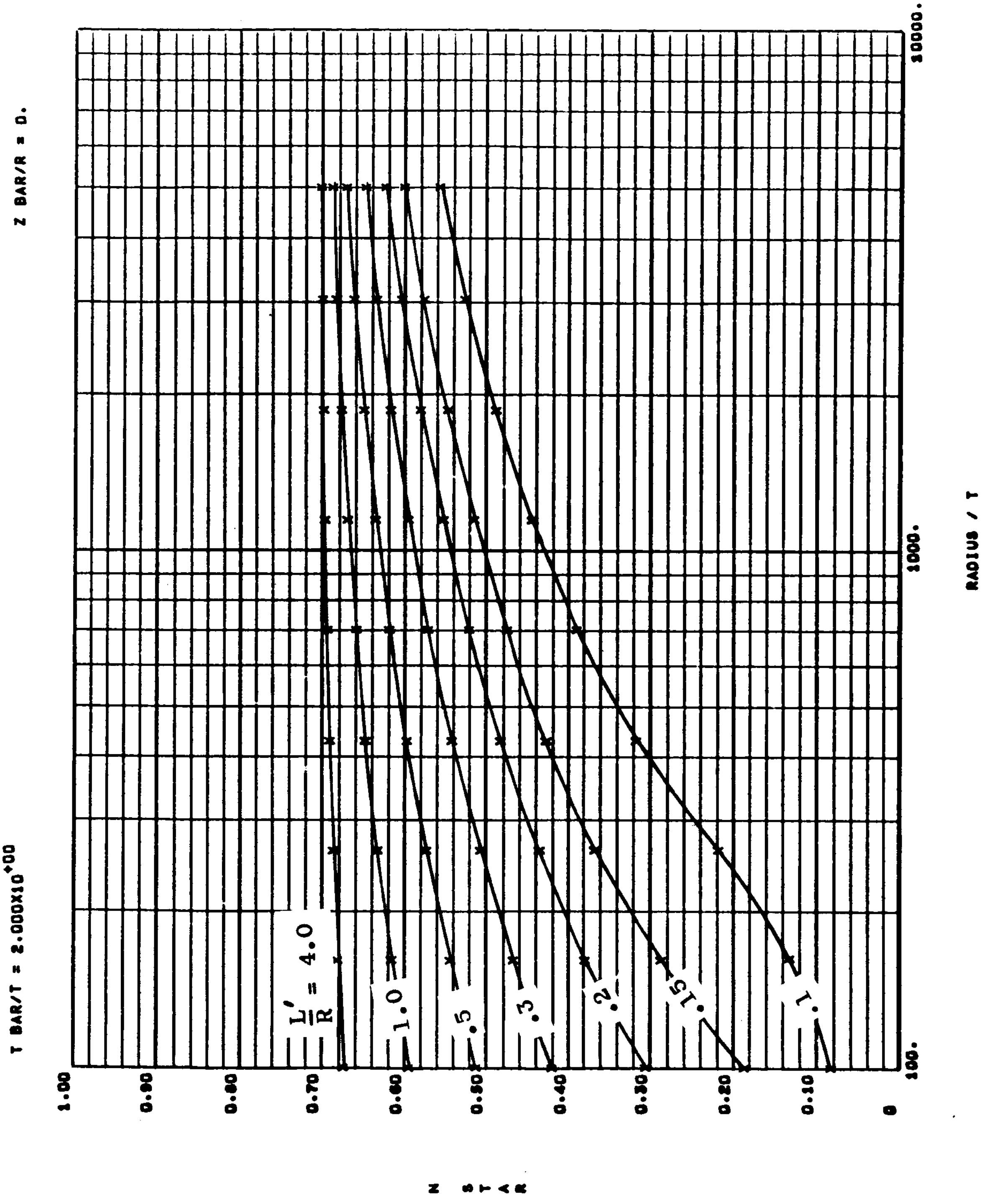
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(p)



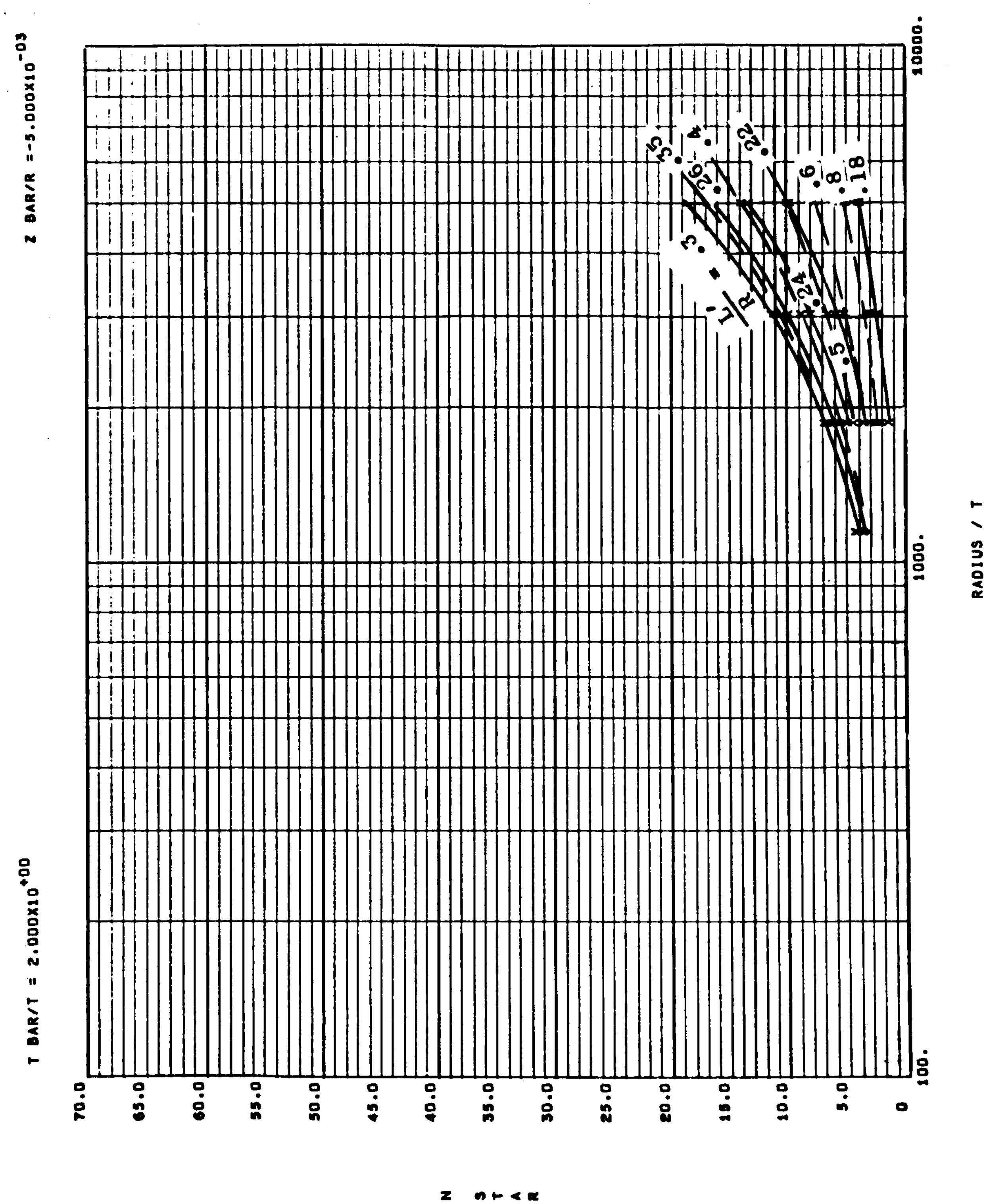
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(p)



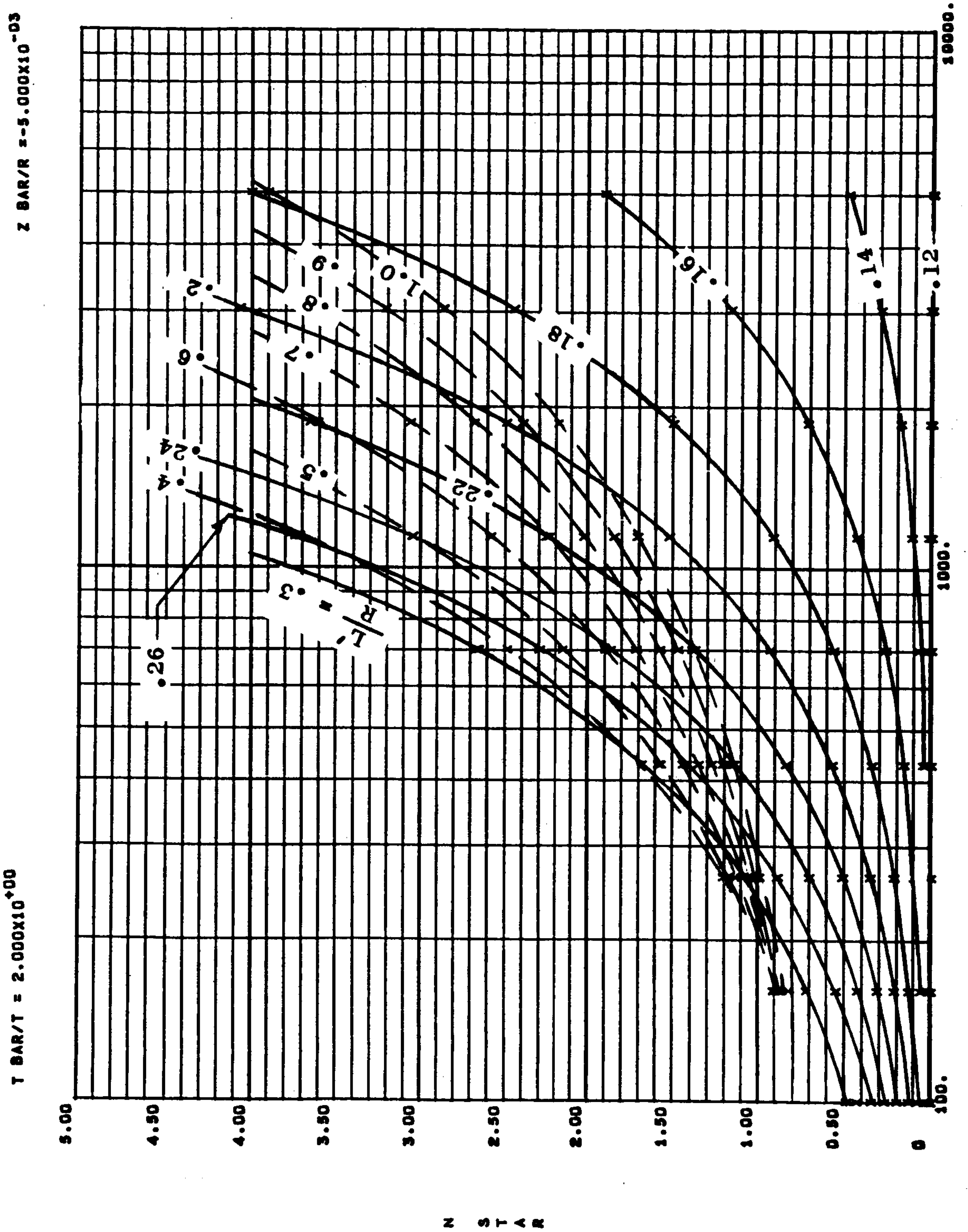
MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(g)



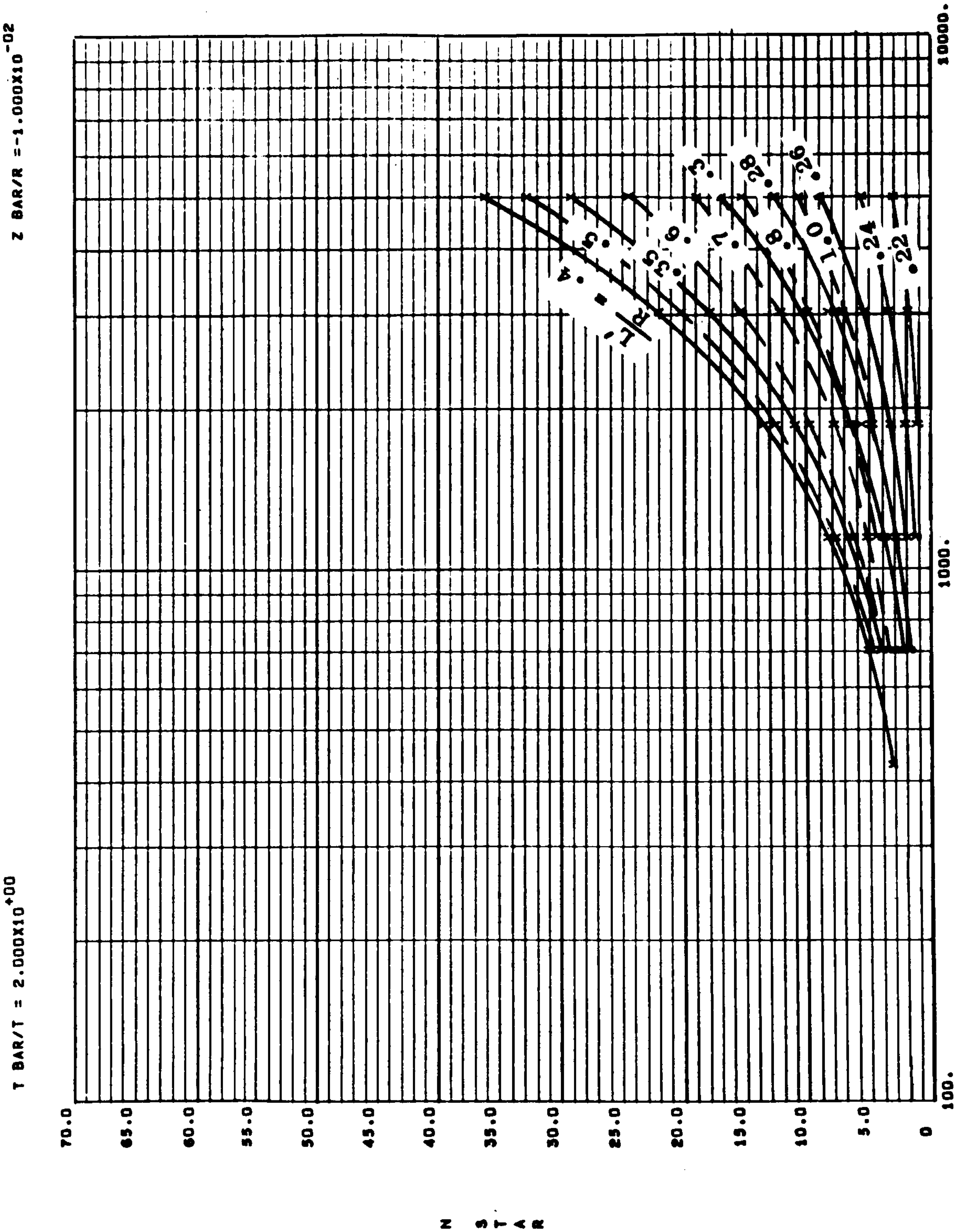
MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(r)



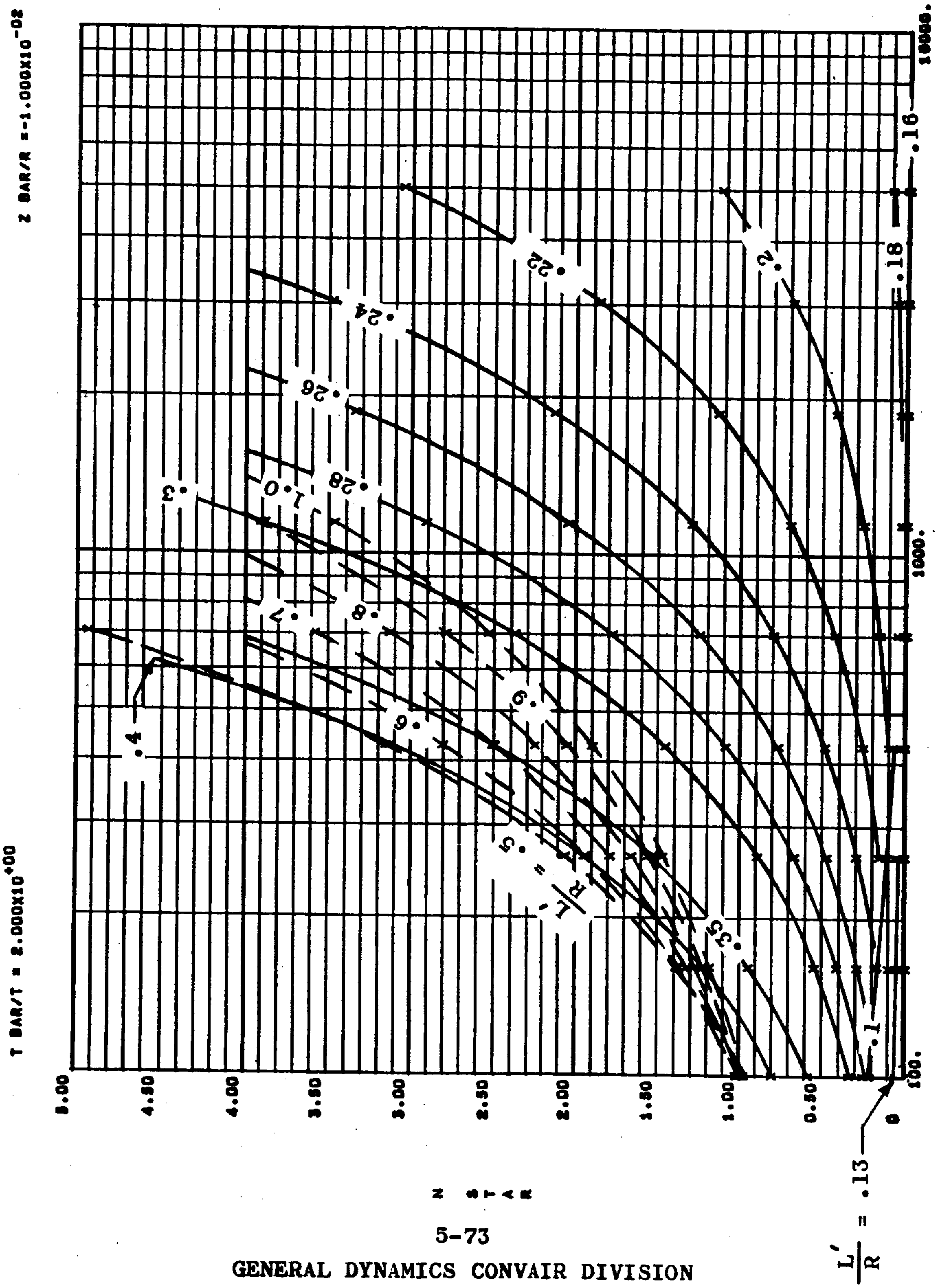
MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(r)



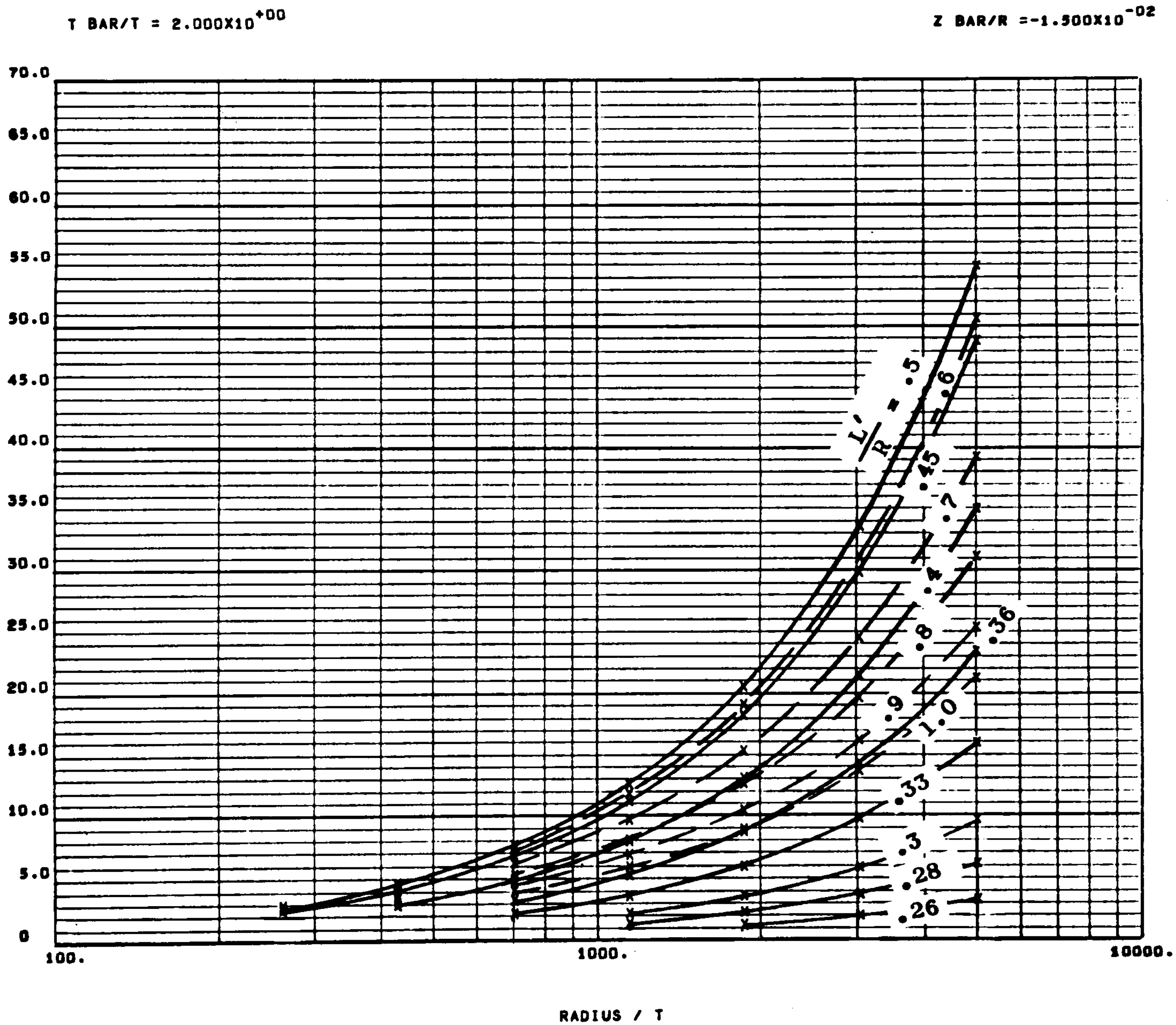
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(s)



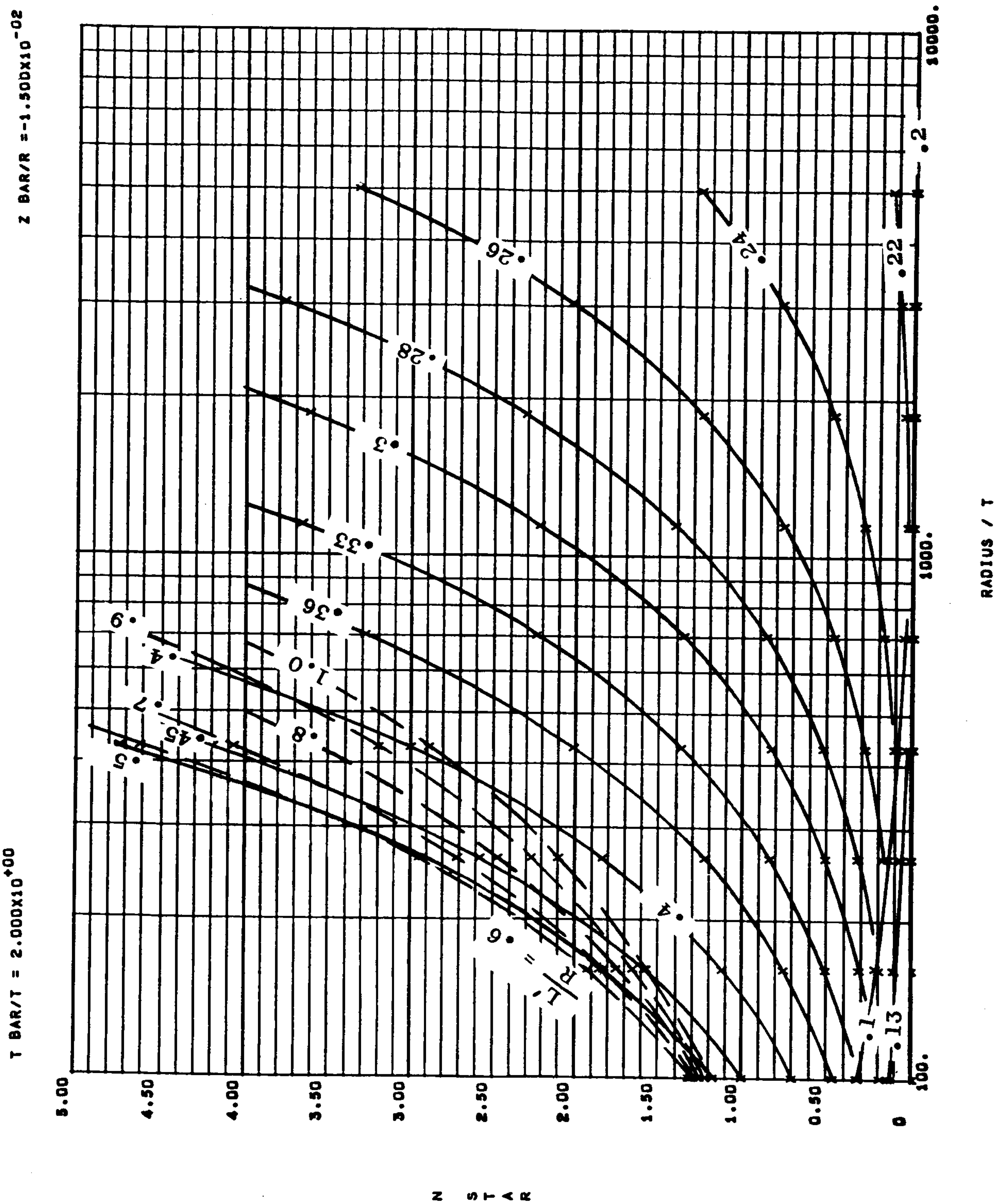
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(s)



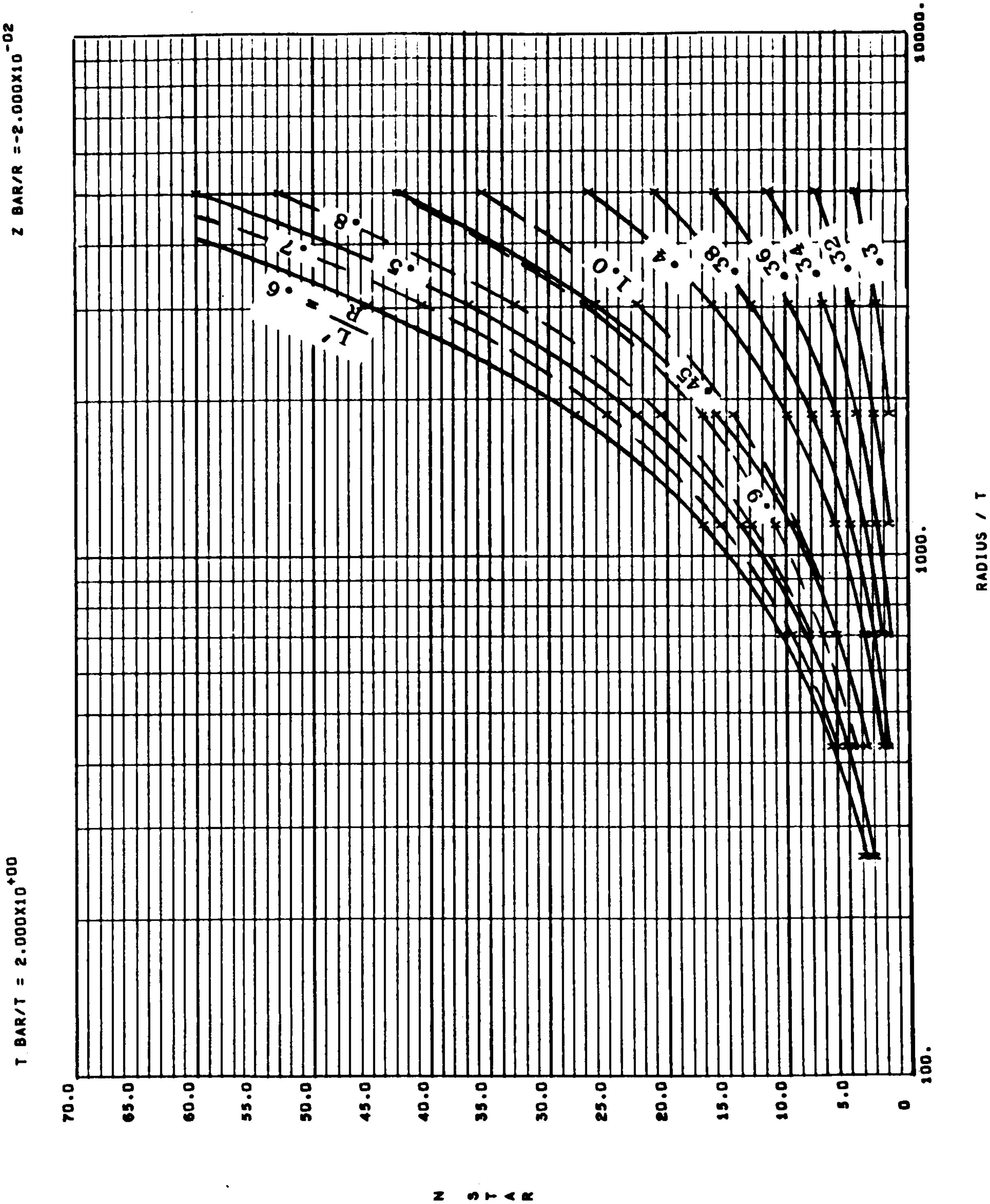
MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(t)



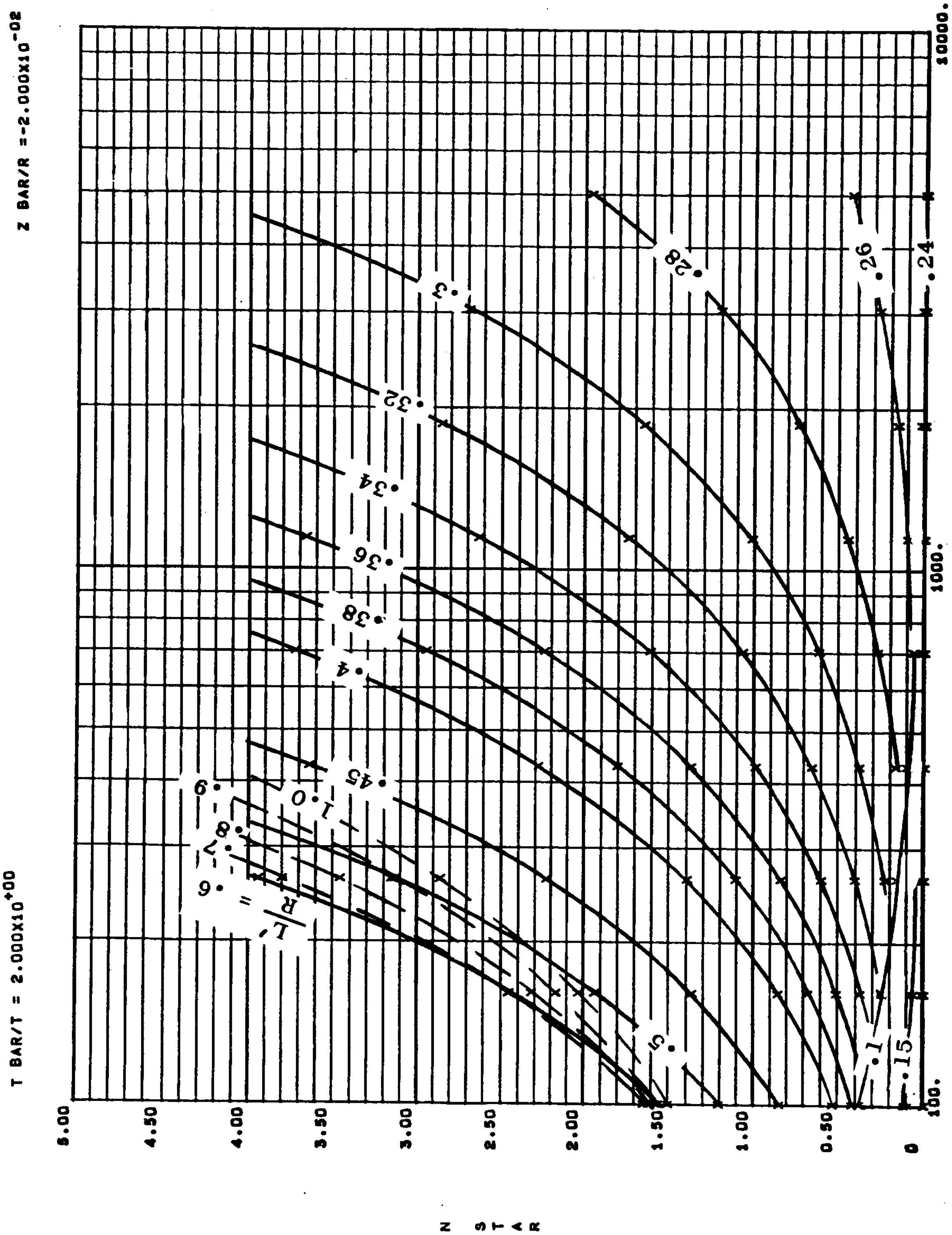
MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(t)



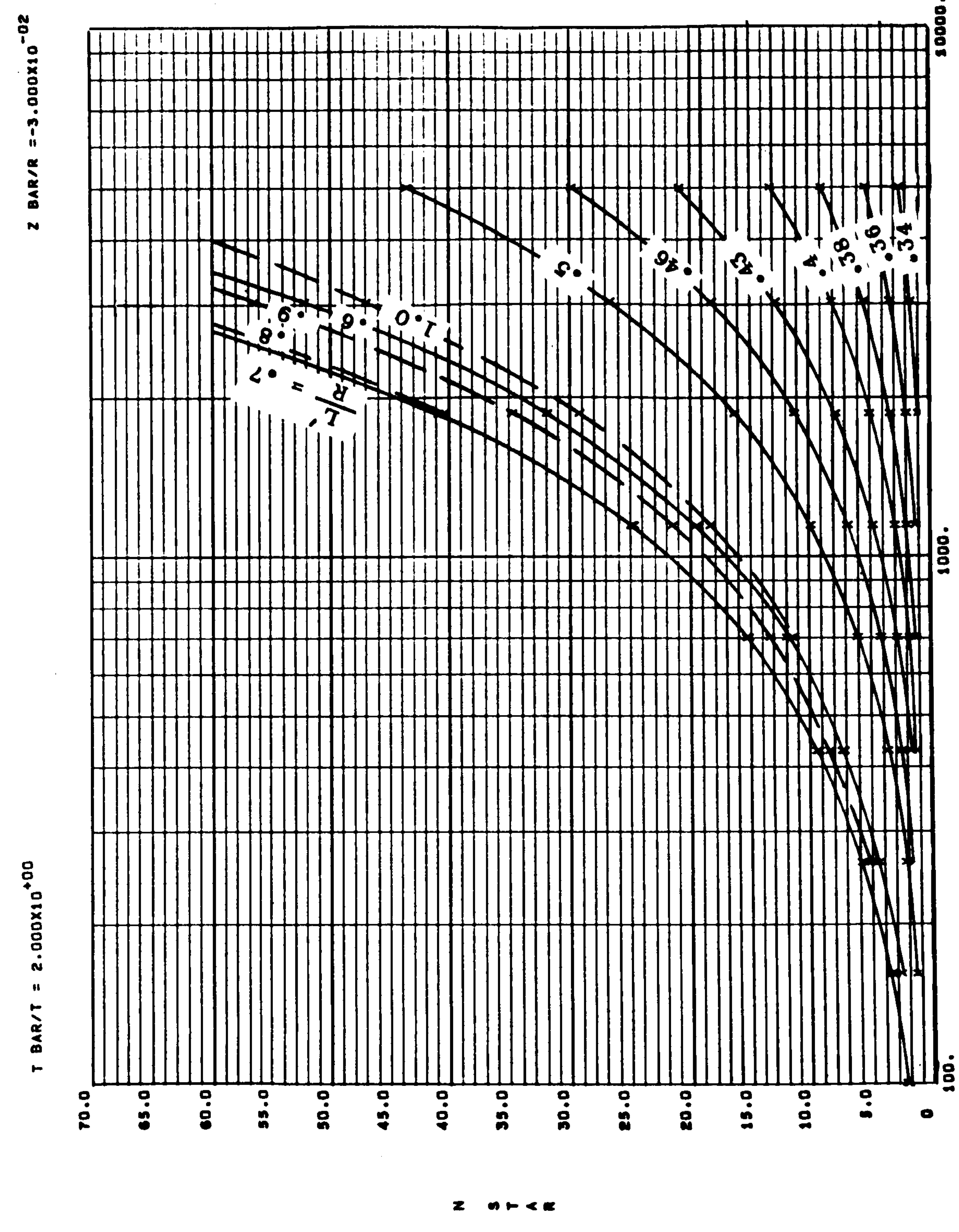
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(u)



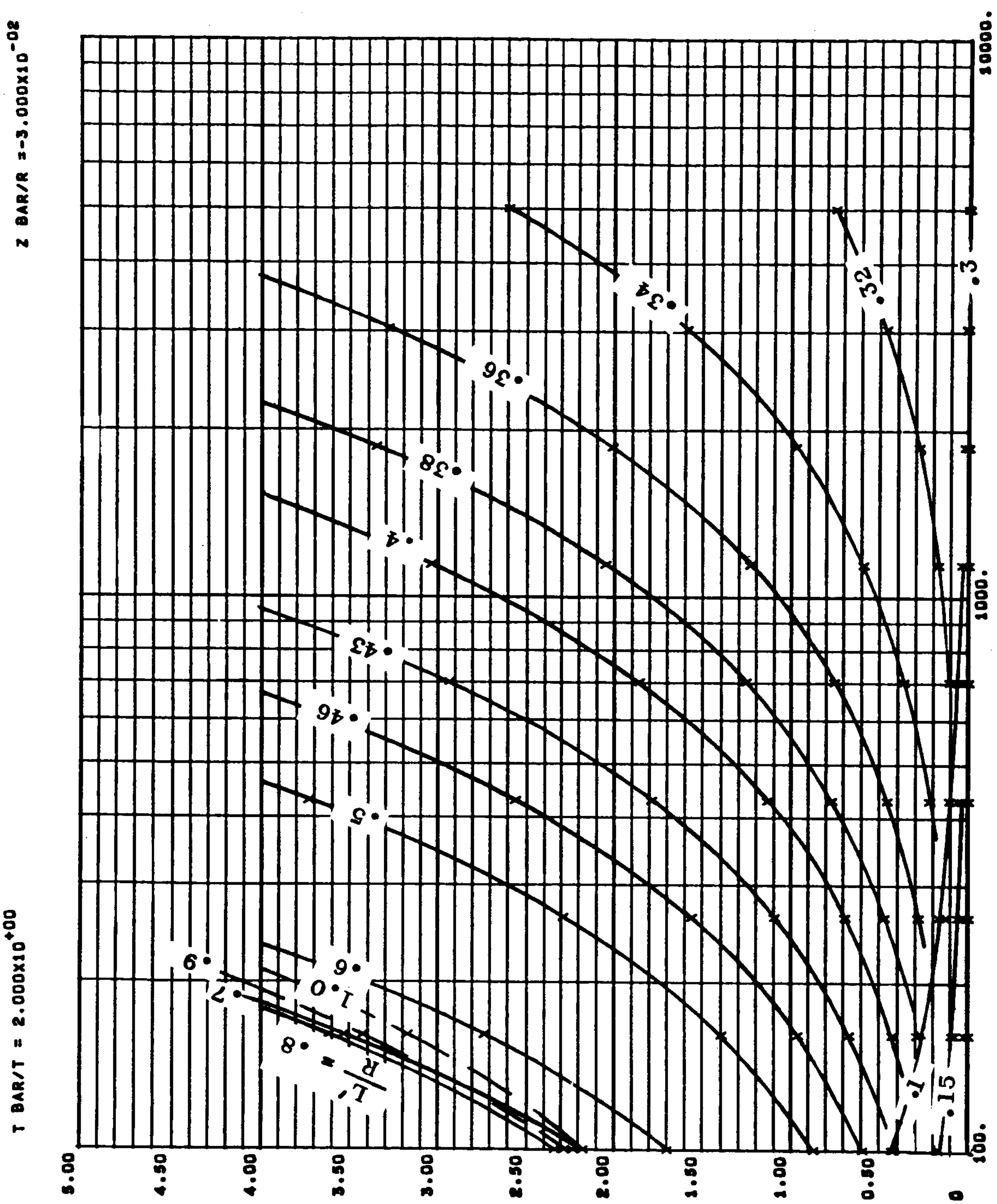
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(u)



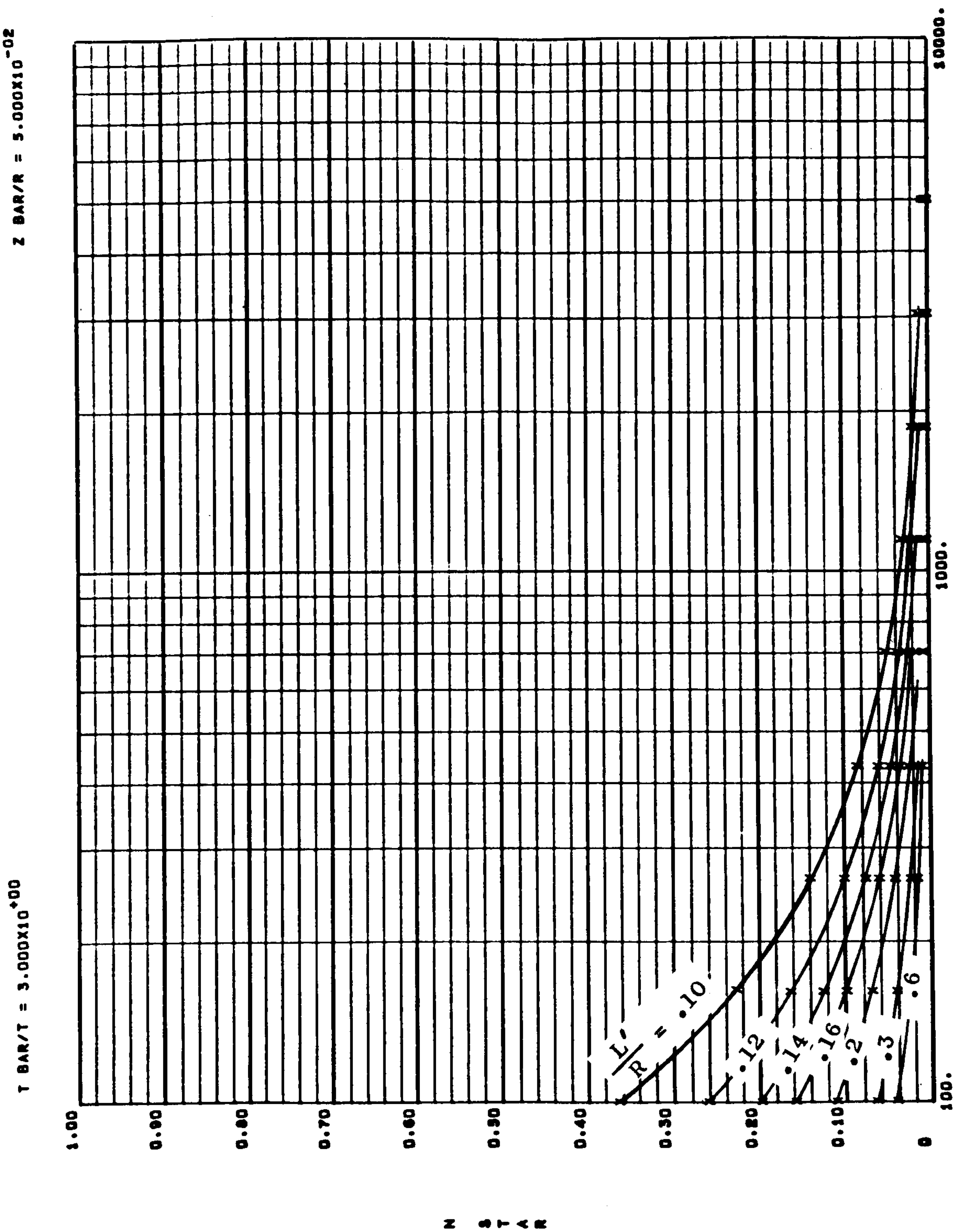
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(v)



MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(v)



MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

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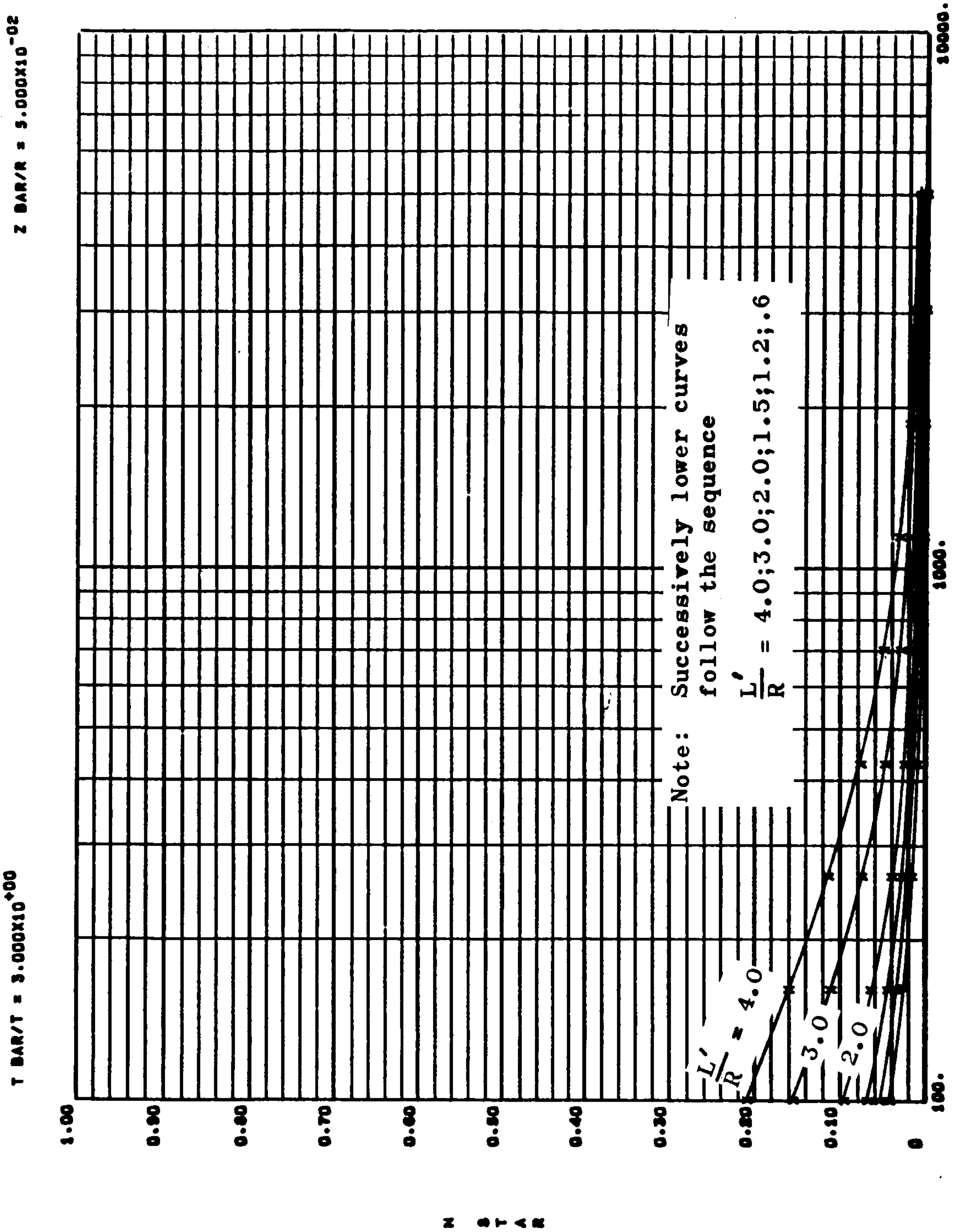
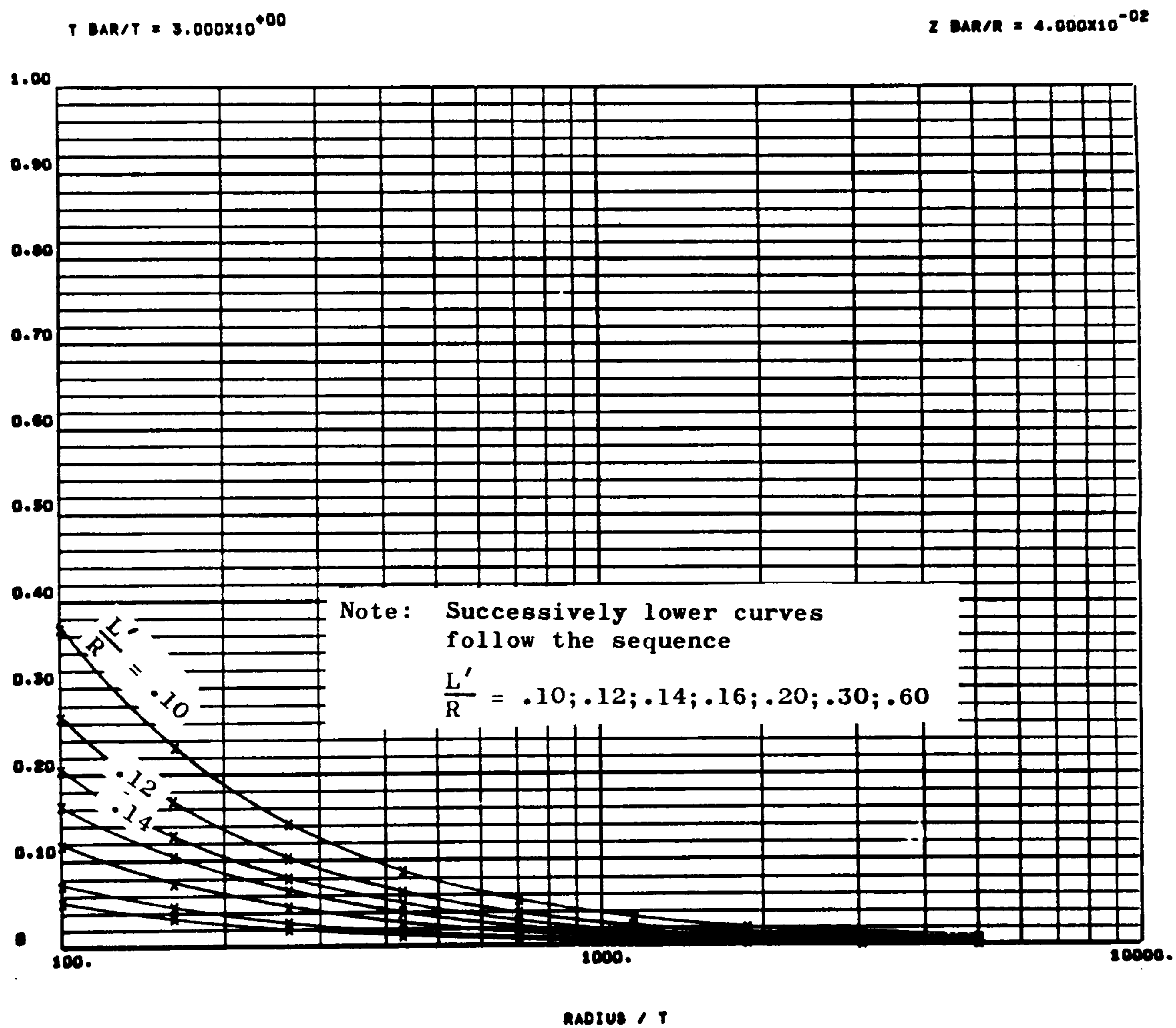


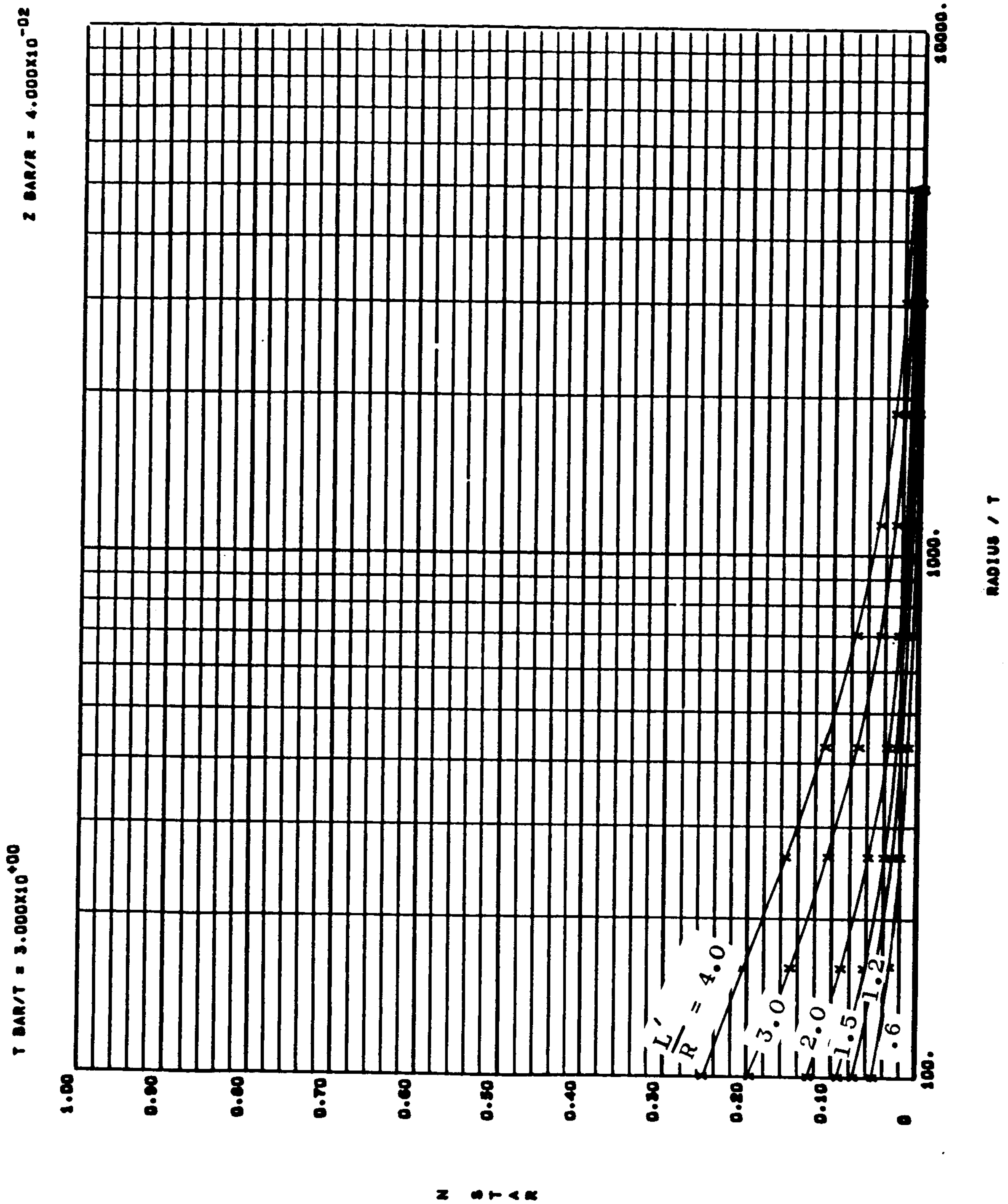
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(w)



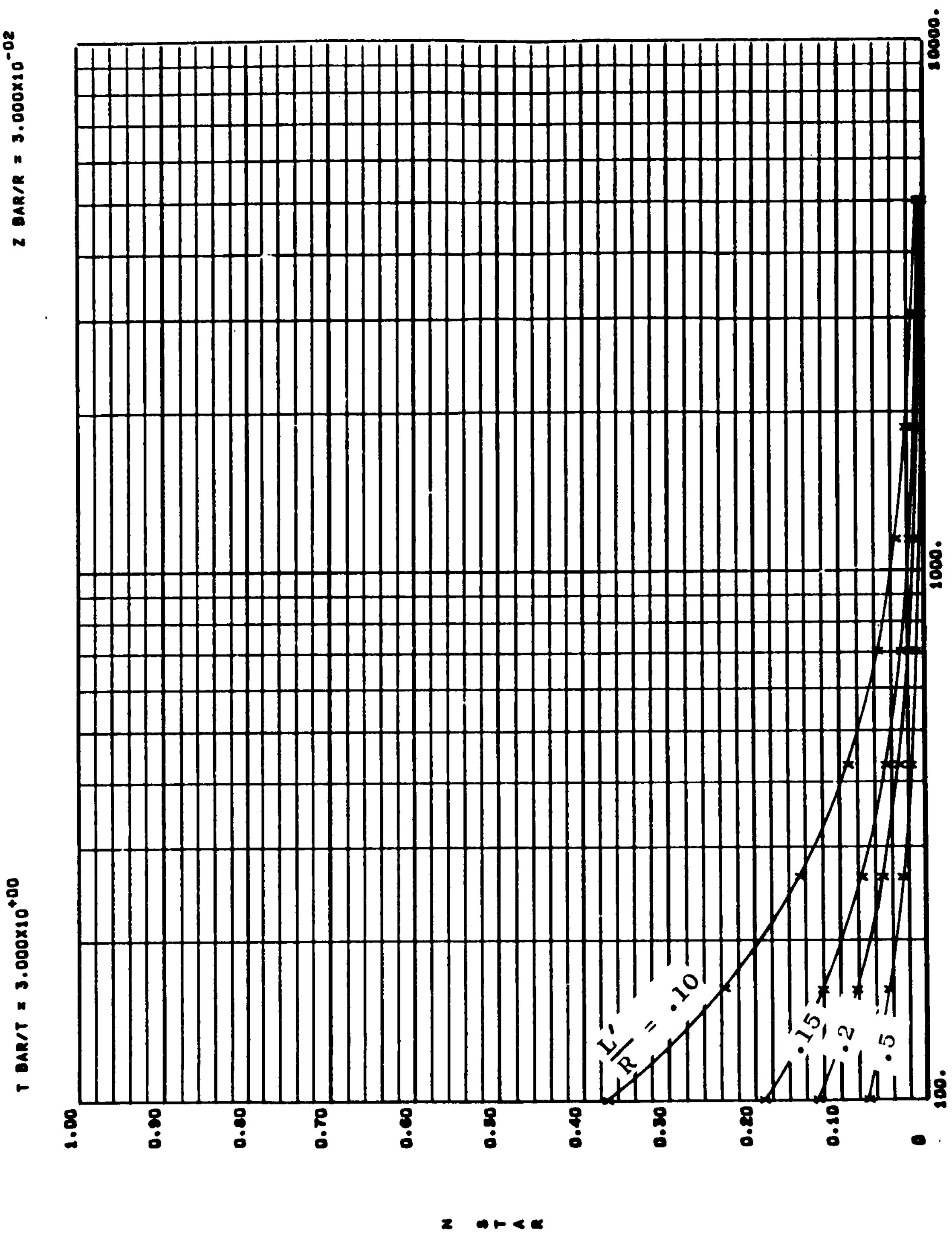
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(x)



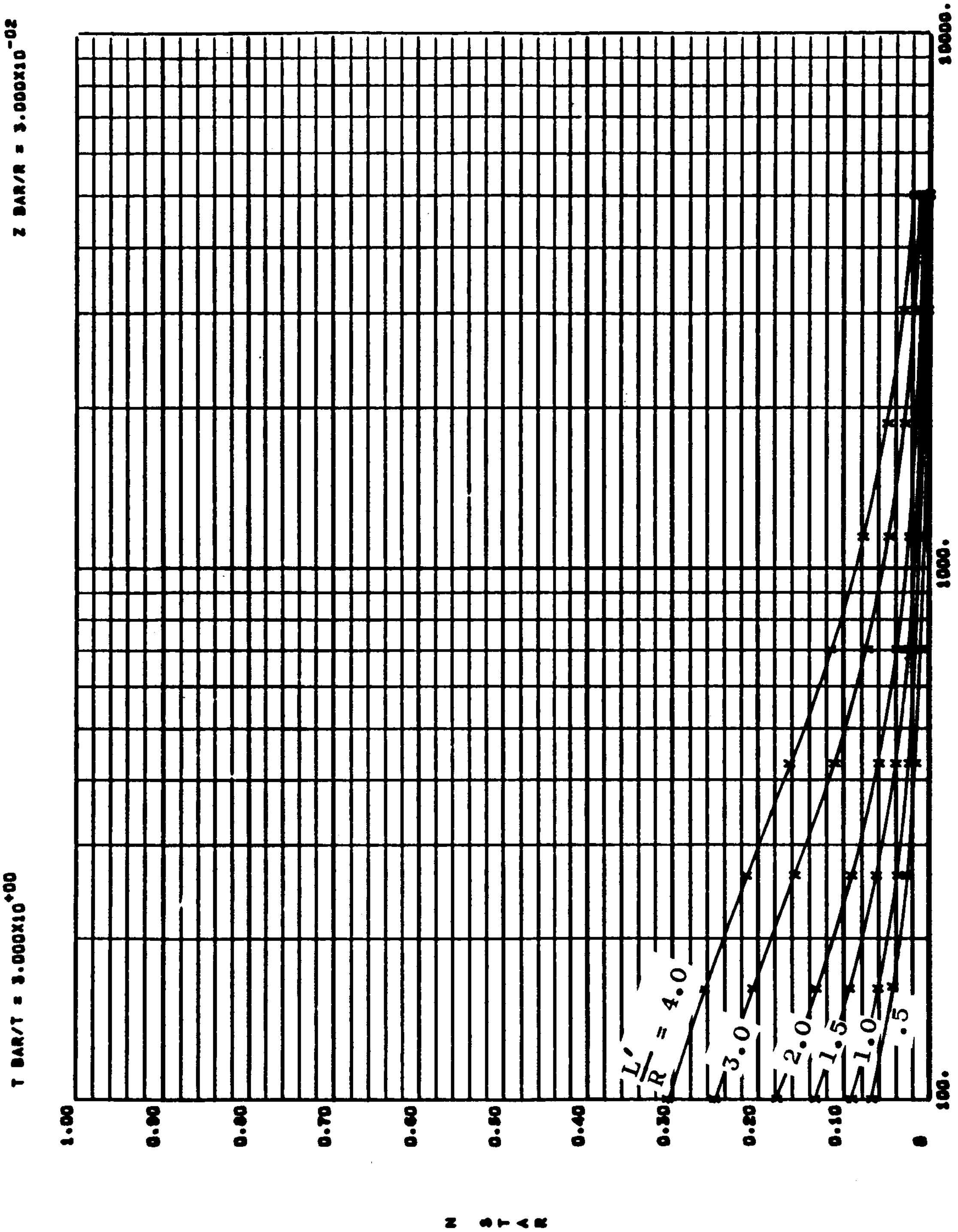
MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(x)



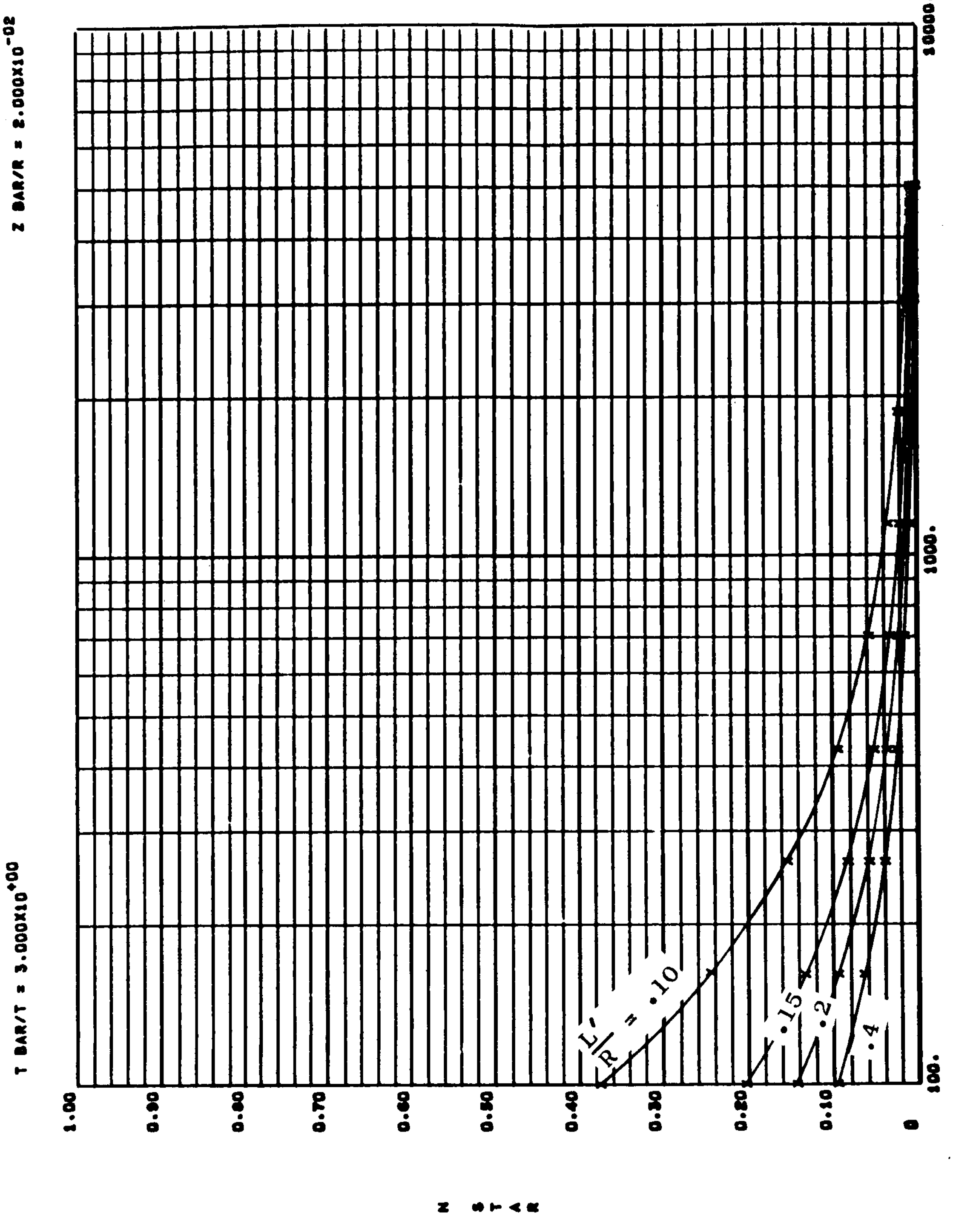
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(y)



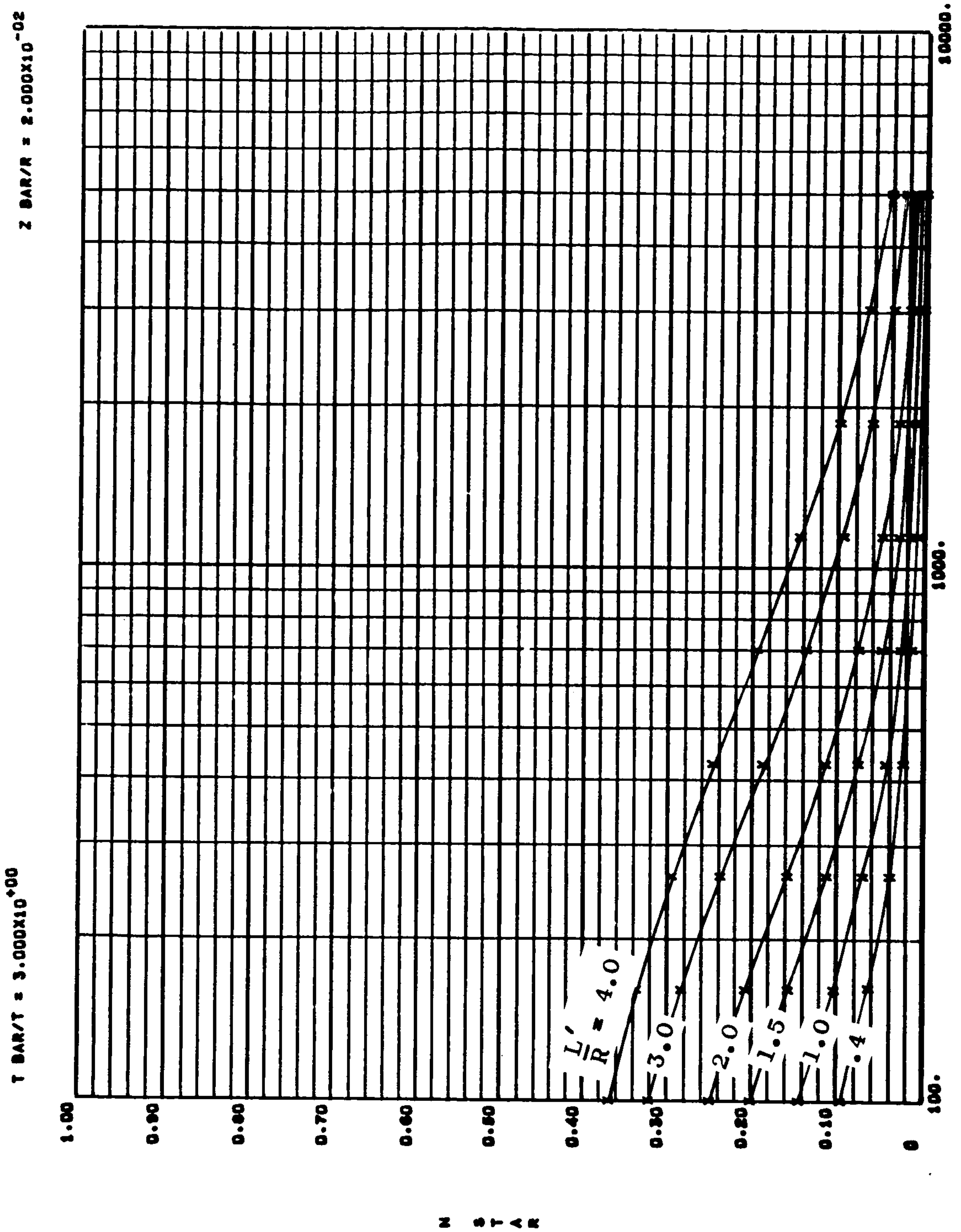
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(y)



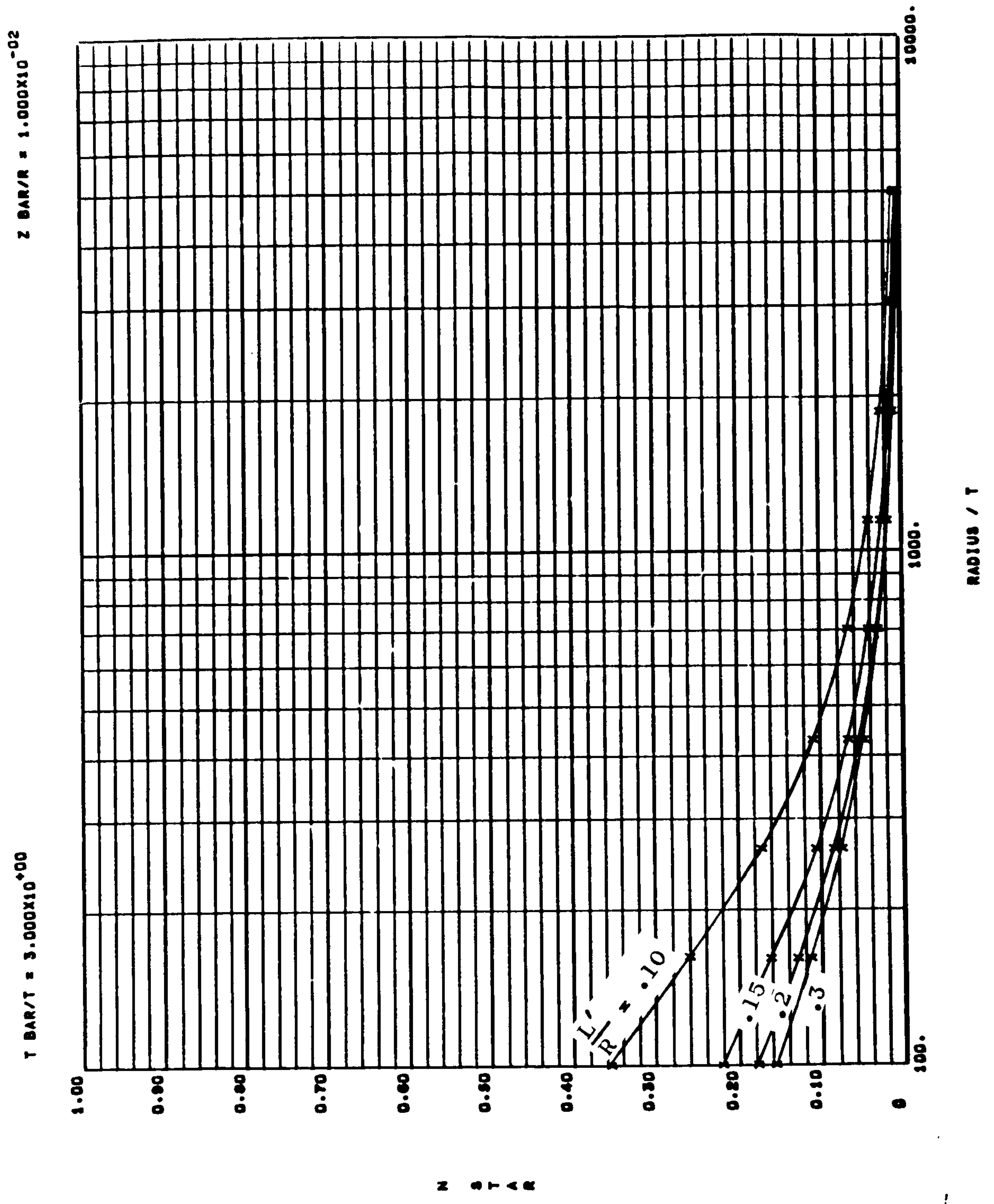
MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(z)



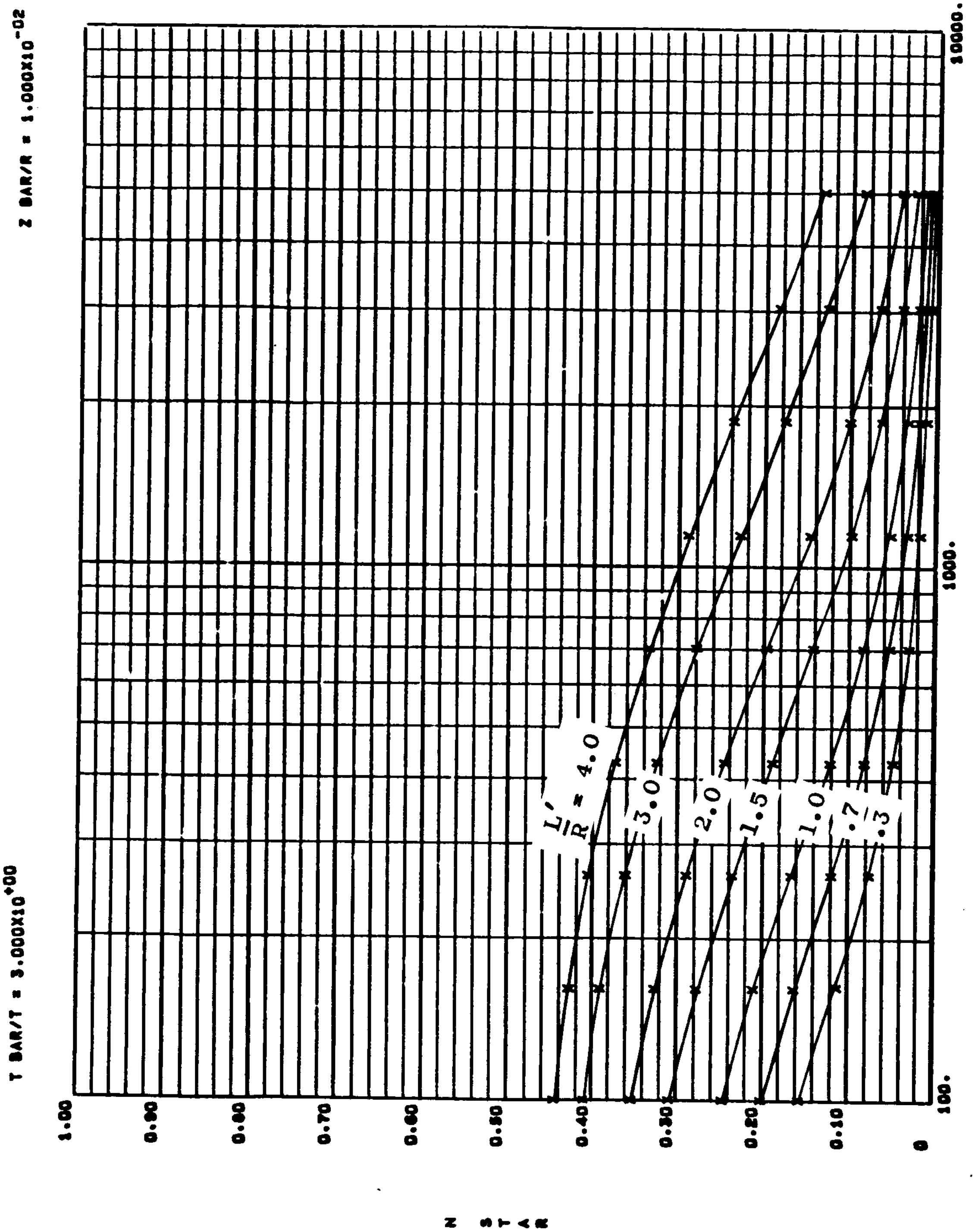
MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(z)



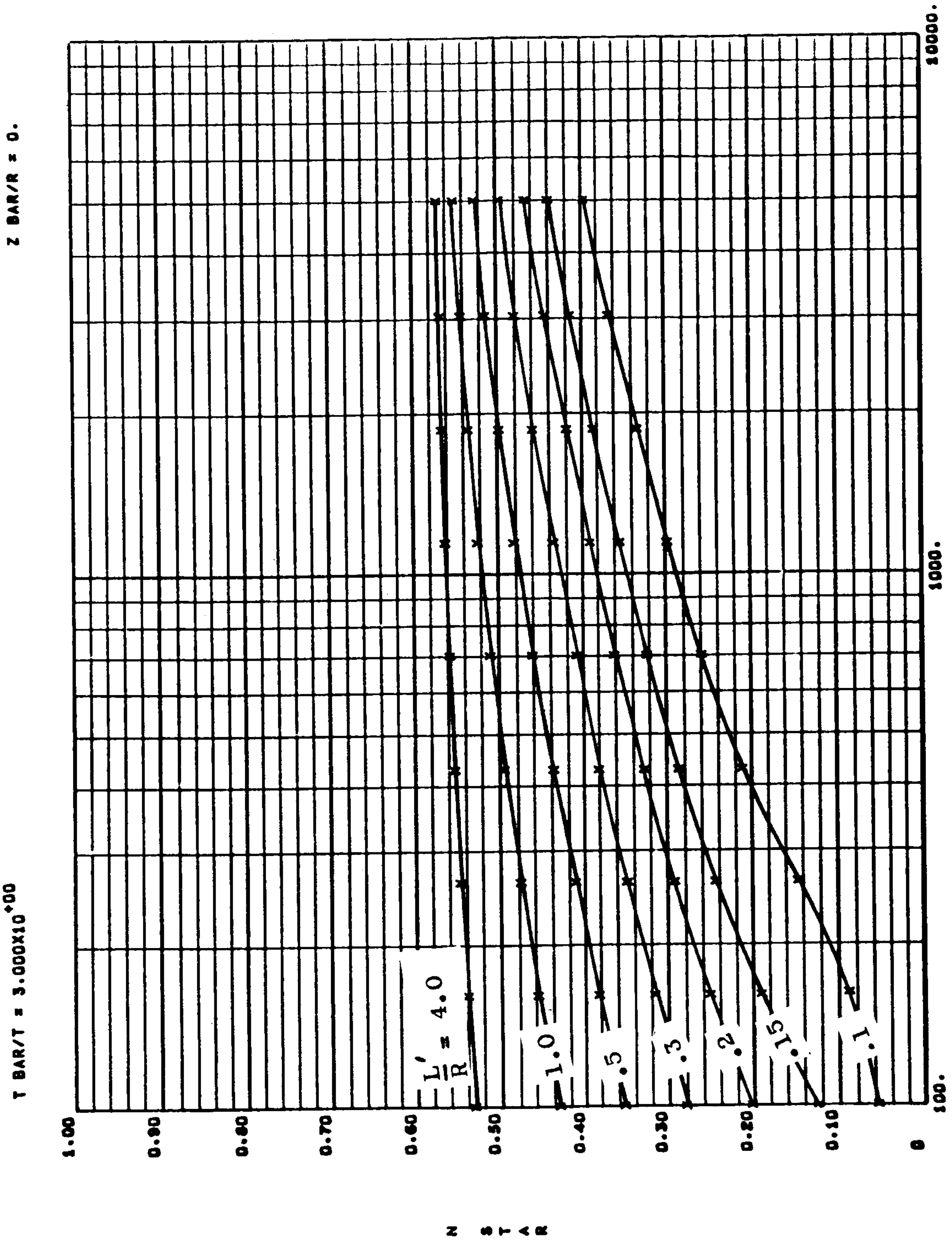
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(aa)



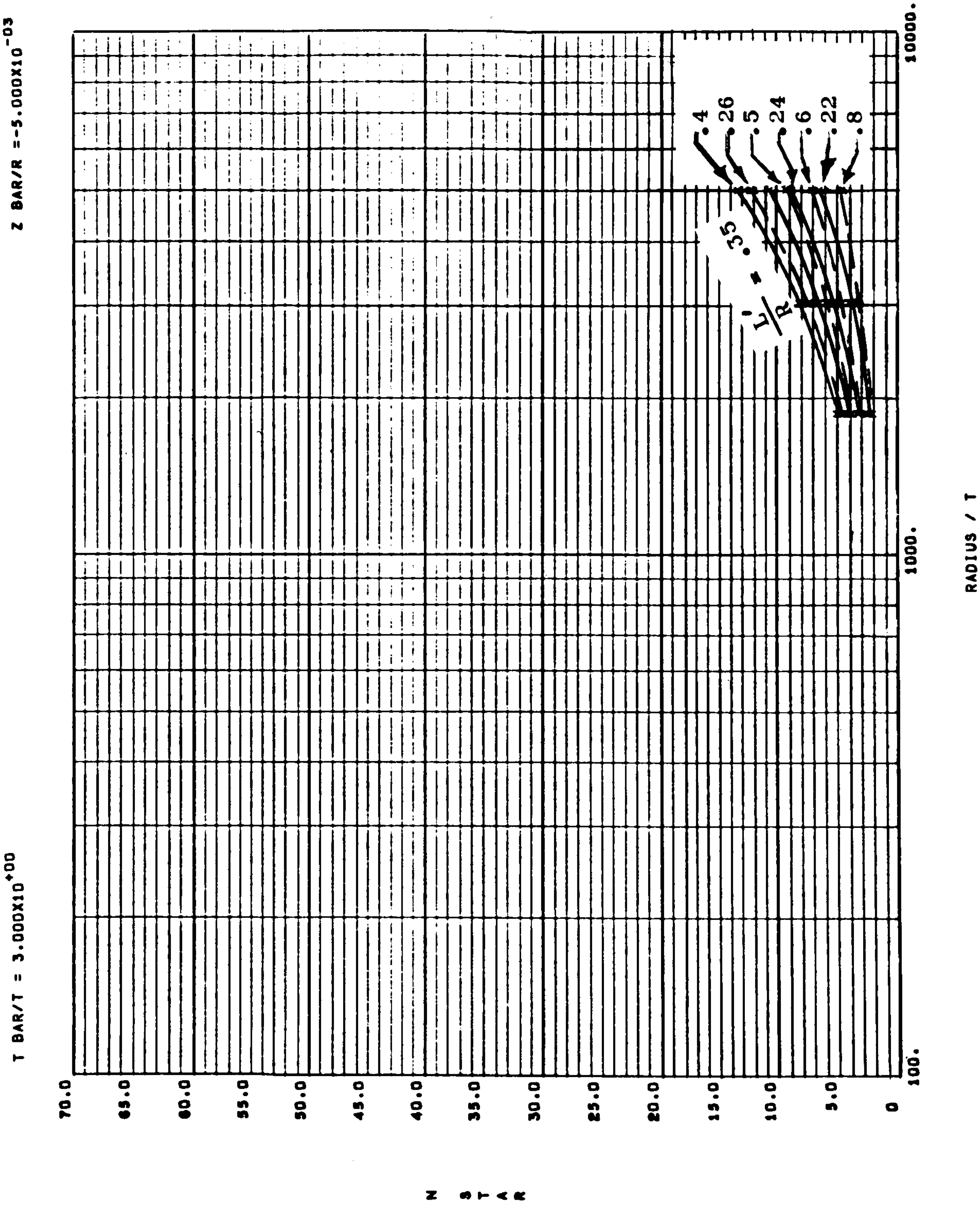
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(aa)



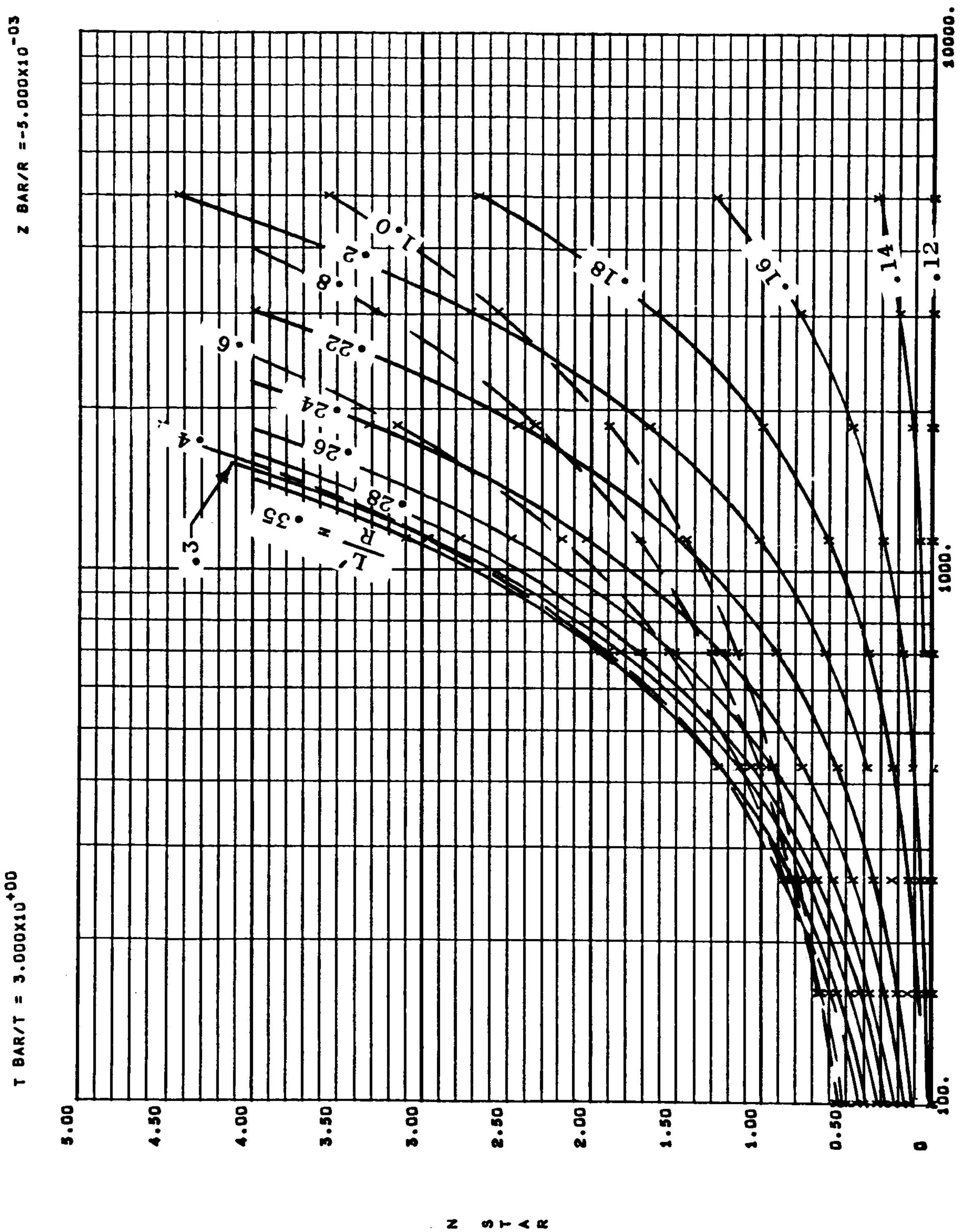
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(bb)



MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(cc)



MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

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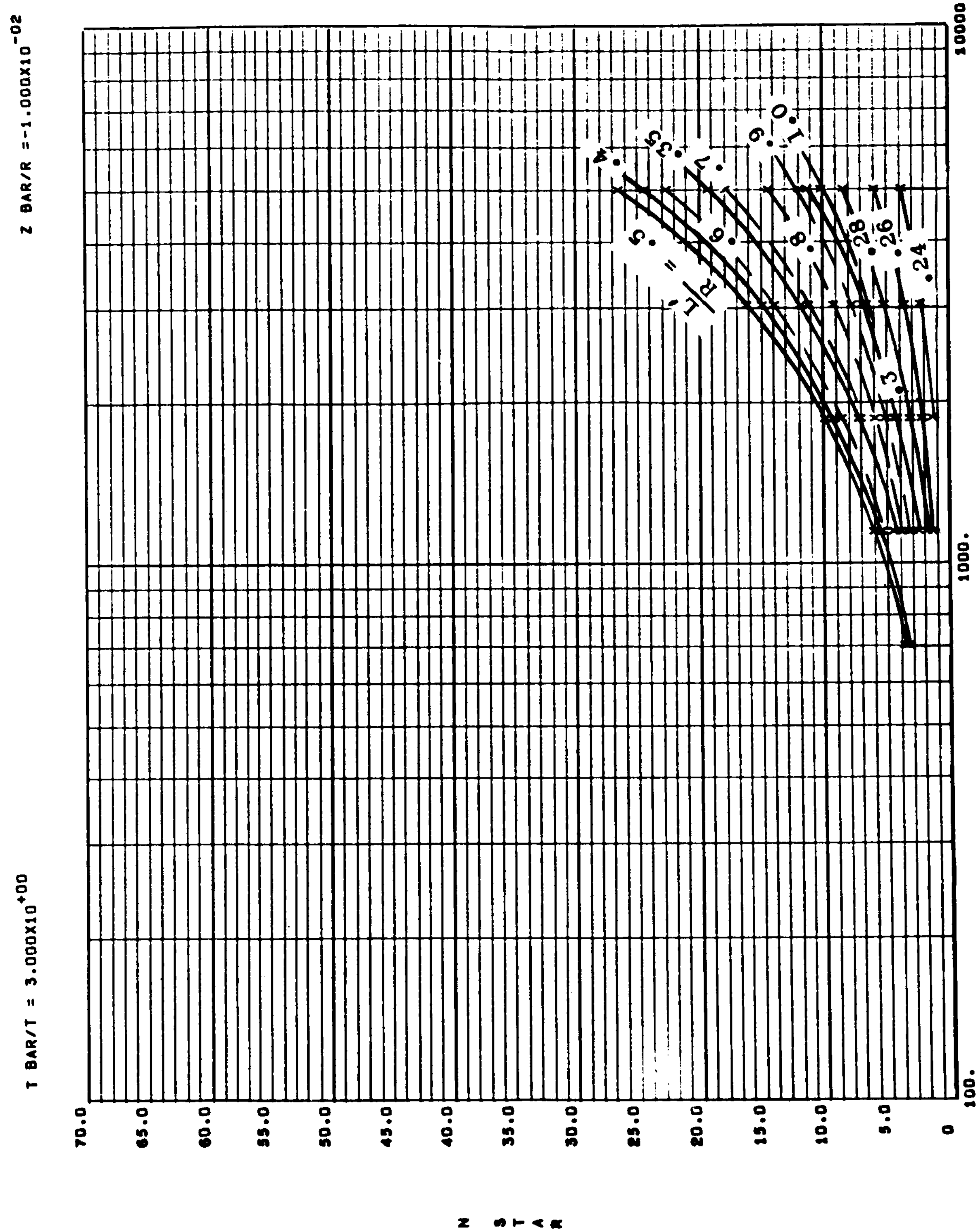
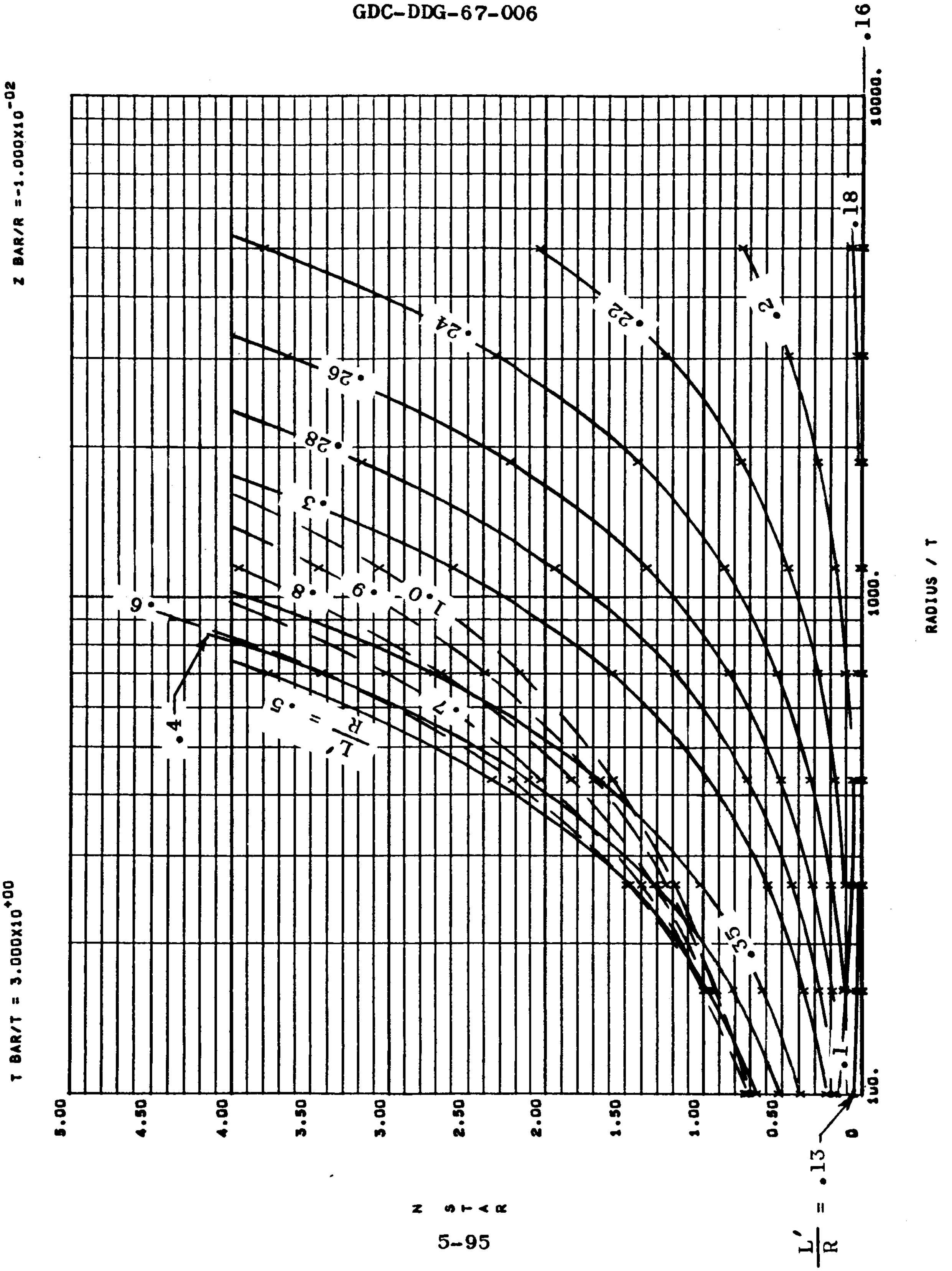


MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(dd)



MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

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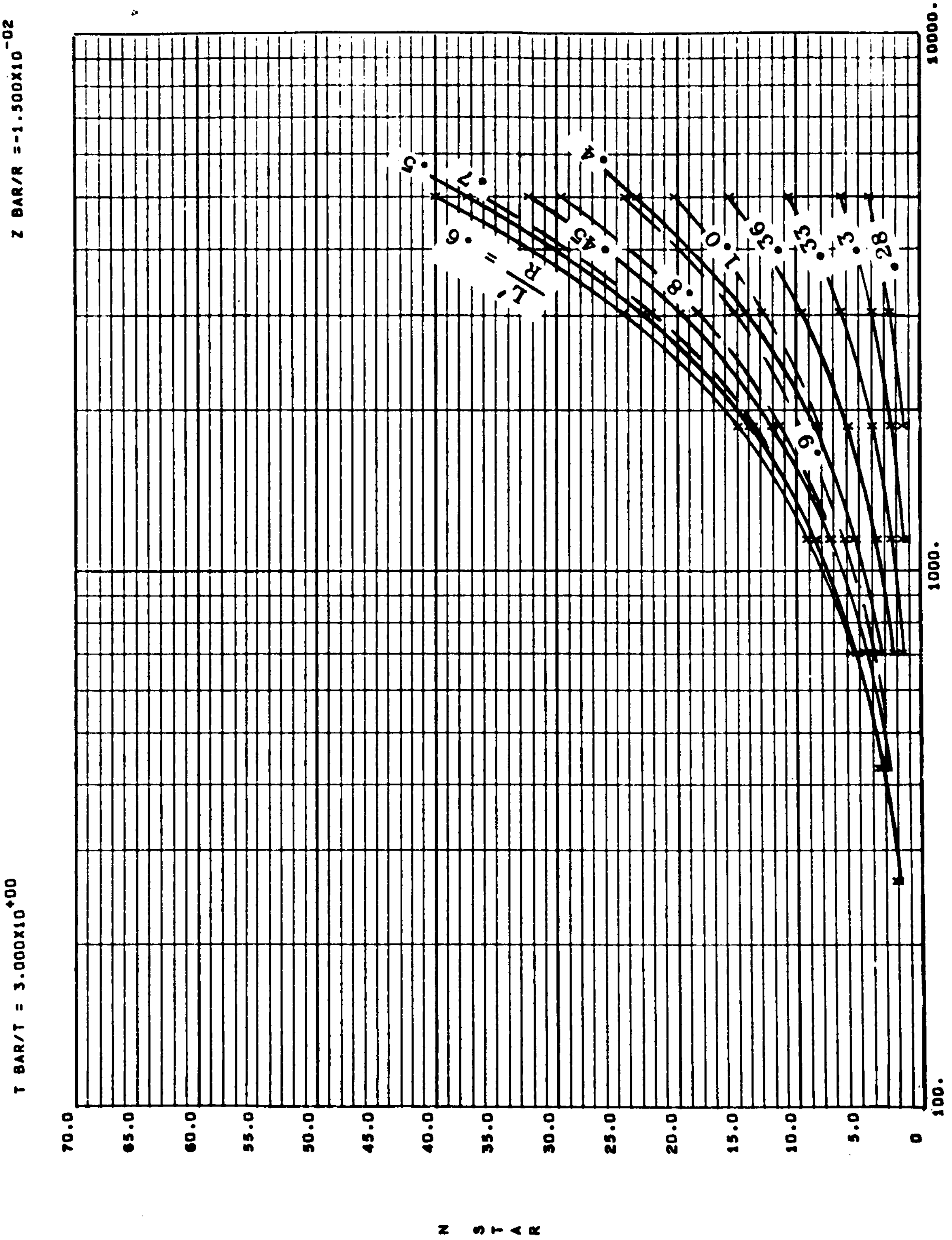
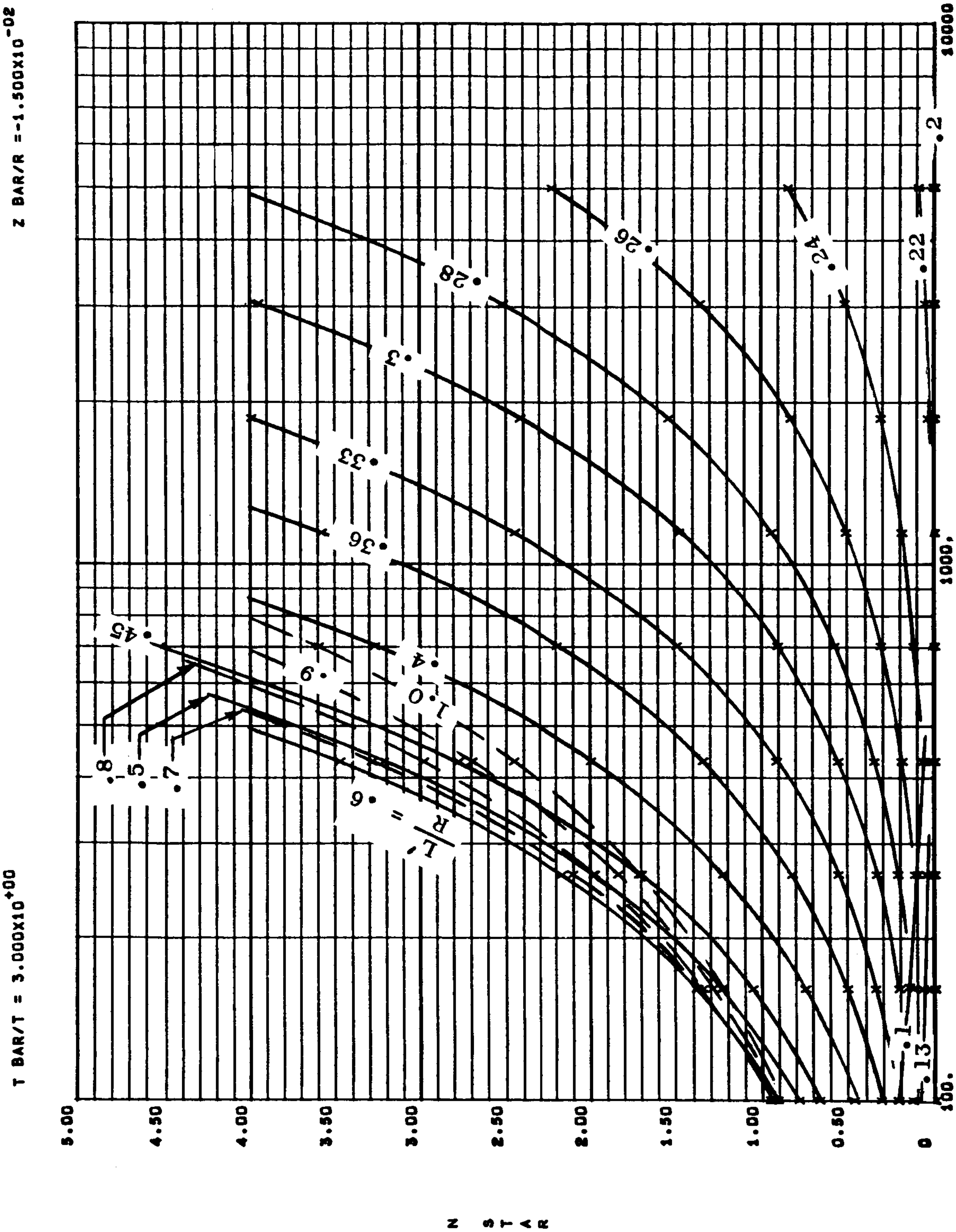


MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(ee)



MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

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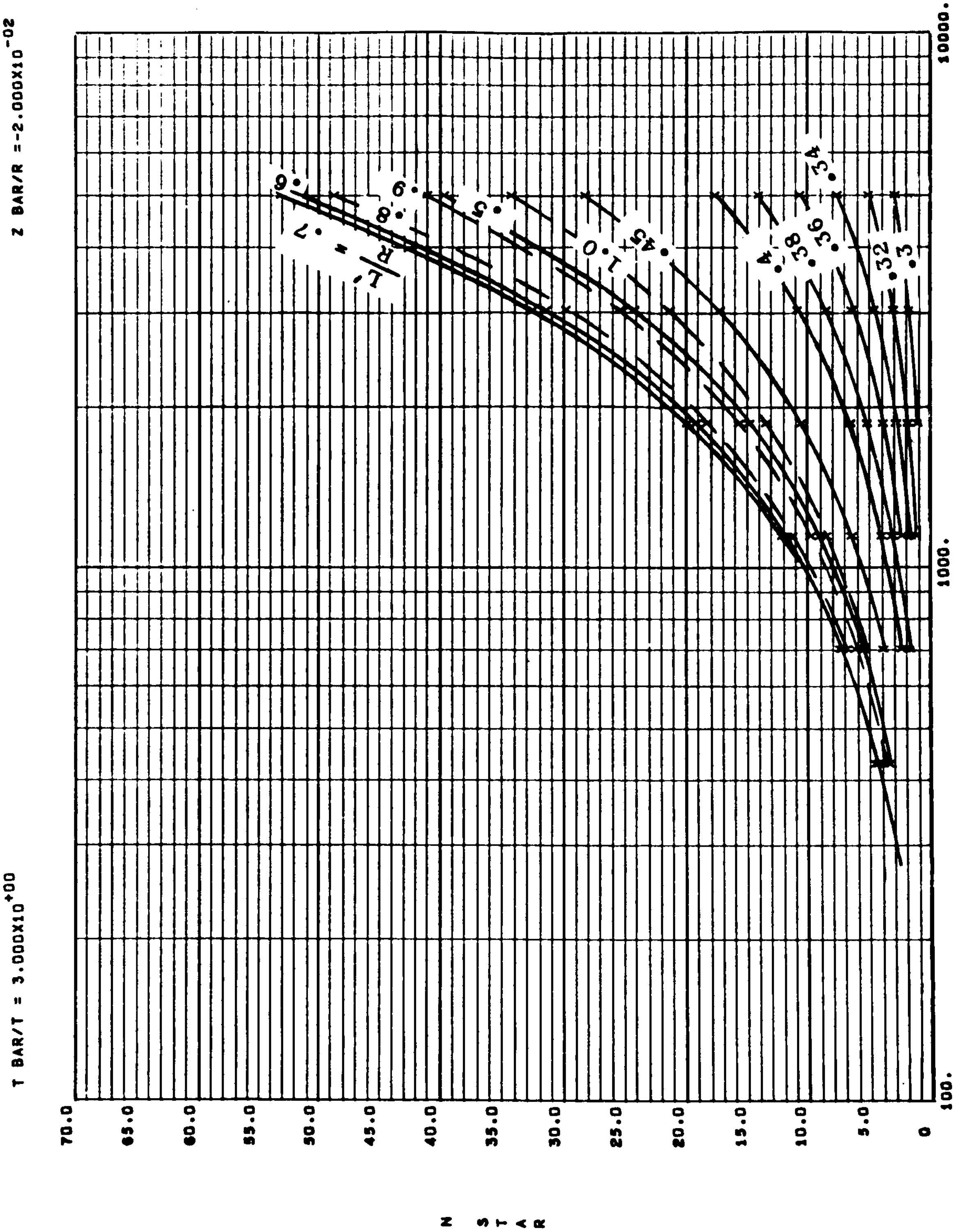
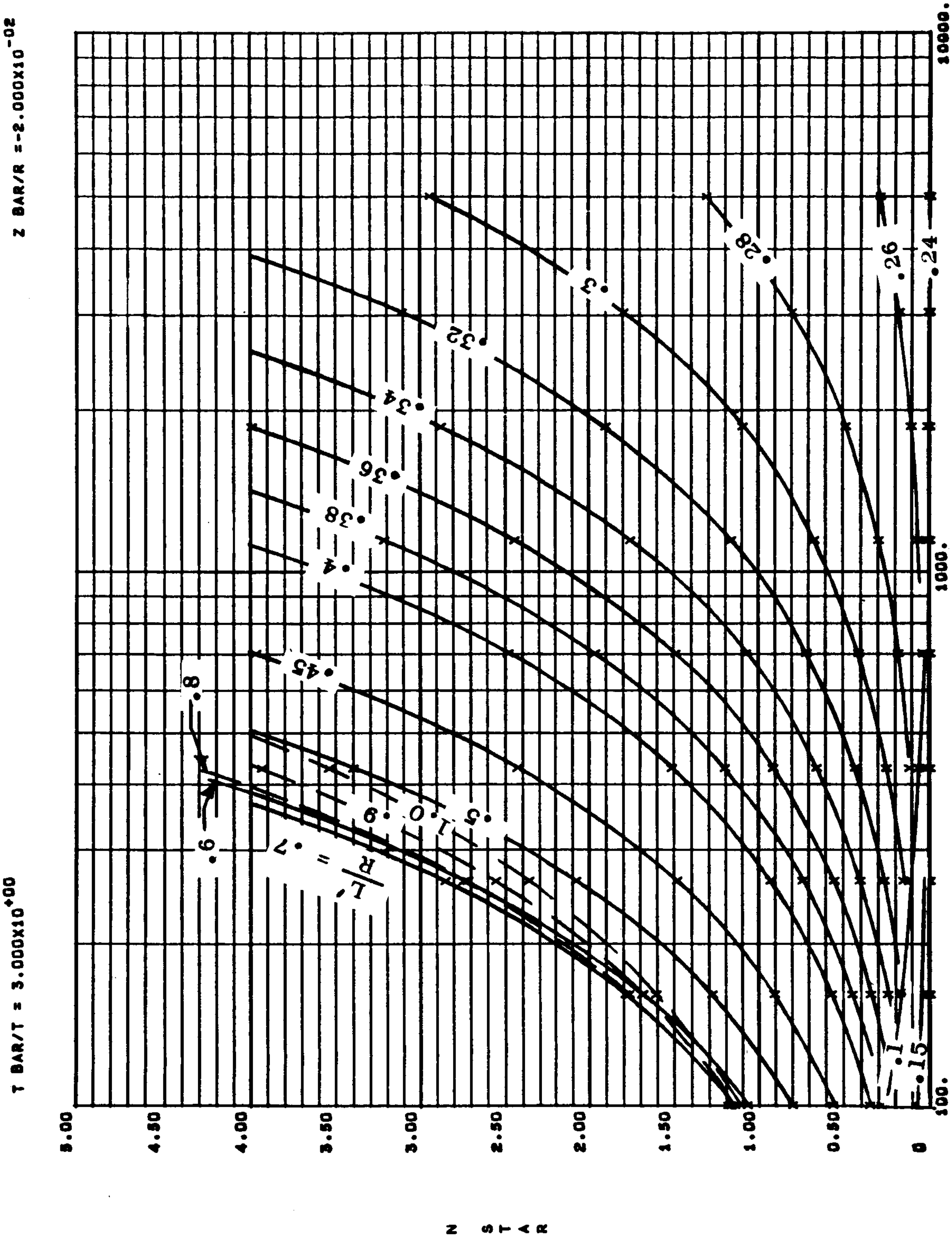


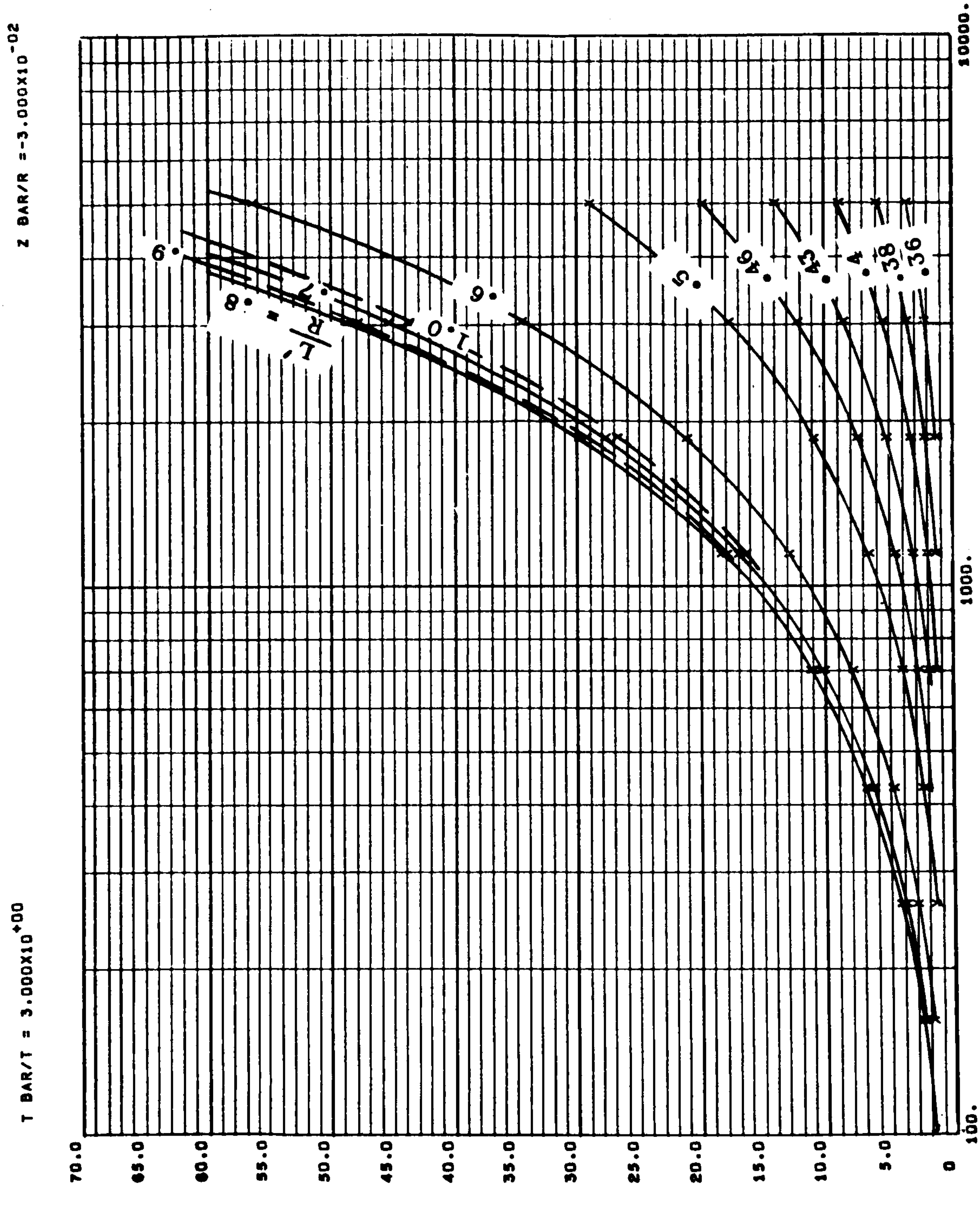
MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(ff)



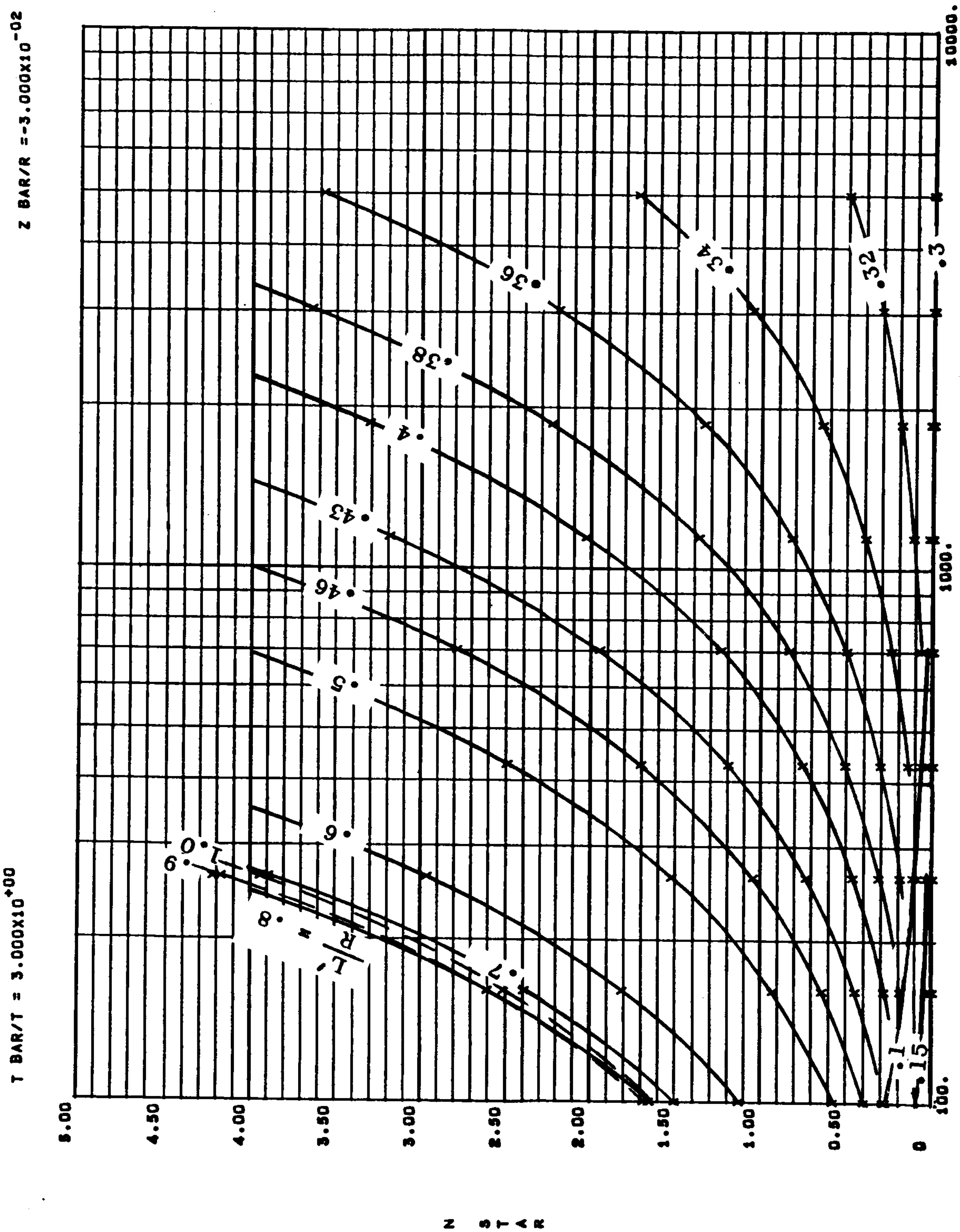
MINIMIZATION FACTOR N STAR FOR
LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(ff)



MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(gg)



MINIMIZATION FACTOR N STAR FOR LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS

Figure 7(gg)

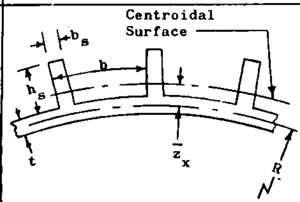
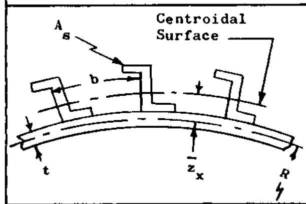
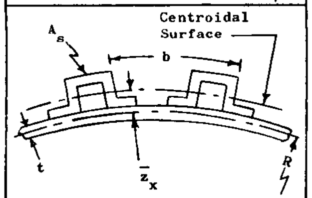
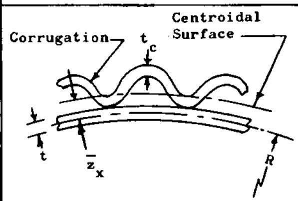
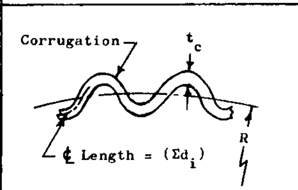
SECTION 6

ELASTIC CONSTANTS

The digital computer program of SECTION 7.2 includes an option which allows for the input of elastic constants and eccentricity coupling constants. This feature was incorporated to provide the engineer with a more flexible analysis method than is given by the curves of SECTION 5.2. However, in order to make use of this capability, one must first compute the values for the various A_{ij}'s, D_{ij}'s, and C_{ij}'s. Recommended formulas for these constants are listed in TABLE X. The tabulated formulas are simplified expressions suitable for practical engineering purposes. To be rigorous, more complicated expressions would be required. All of the given formulas apply only where the behavior is elastic. For cases where the buckling stress exceeds the proportional limit of the stress-strain curve, it is recommended that E and G be replaced by E_{tan} and G_{tan}, respectively. To fully understand TABLE X, it is helpful to note that the A_{ij}'s and D_{ij}'s arise out of mathematical integrations involving the distribution of the composite wall material about the appropriate cylindrical centroidal surface. Note that the centroidal surface has curvature of its own. Therefore, the related material distribution is equivalent to that which exists about the centroidal plane of the flat plate obtained by unfolding the composite circular shell wall into a flat configuration. All influences of curvature, in this regard, are inherent in the basic shell equations into which the A_{ij}'s and D_{ij}'s are substituted.

Table X applies only to cases where no buckling of the isotropic skin panels and no local buckling of the stringers occurs prior to overall instability. In addition, it is assumed that the stringers are spaced sufficiently close together to justify the assumption that all of the skin material is fully effective. In this volume, it is recommended that the shell-type components of equations (2-24) and (2-25) be ignored ($N^* = 0$) whenever the overall instability is preceded by buckling of the isotropic skin panels. The total strength of the longitudinally stiffened cylinder then reduces to the conventional wide-column value established by the well-known Euler-Johnson expressions. However, one might encounter situations where there is no buckling of the isotropic skin panels but where local buckling of the stringers occurs prior to the overall instability. In these cases the shell component should be retained in the analysis. It then becomes necessary to employ effective-width concepts to modify the information given in TABLE X and the notes which follow it. Similar modifications would be required where the stringer spacings were too great to justify the assumption of fully effective skins.

TABLE X - Recommended Formulas for the Elastic Constants and Eccentricity Coupling Constants of Longitudinally Stiffened Circular Cylinders

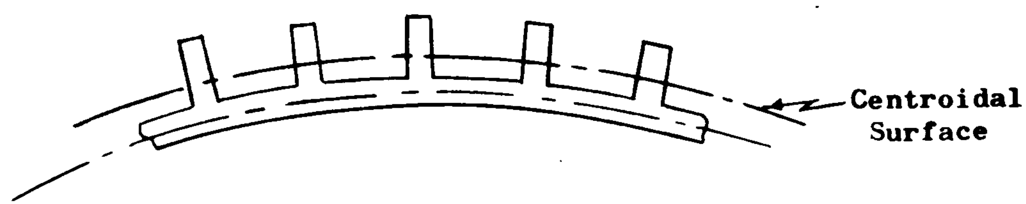
CASE	CONFIGURATION	$\bar{t}_x$	A_{11}	A_{22}	A_{12}	A_{33}	D_{11}	D_{22}	D_{12}	D_{33}	C_{11}	C_{12}
A		$\left(\frac{b_s h_s}{b} + t\right)$	$\frac{1}{E\bar{t}_x}$	$\frac{1}{Et}$	$\frac{-\nu}{E\bar{t}_x}$	$\frac{1}{Gt}$	$E\bar{I}_x$	$\frac{Et^3}{12(1-\nu^2)}$	νD_{22}	$\frac{Gt^3}{12}$	$\bar{z}_x$ (Positive for internally stiffened; Negative for externally stiffened)	$-\nu\bar{z}_x$
		$\left(\frac{A_s}{b} + t\right)$										
												
	ETC.											
B		$\left[\left(\frac{\Sigma d_i}{2\pi R}\right) t_c + t\right]$	$\frac{1}{E\bar{t}_x}$	$\frac{1}{E\left[\frac{t_c}{\left(\frac{\Delta_x}{\delta_x}\right)} + t\right]}$	$\frac{-\nu}{E\bar{t}_x}$	$\frac{1}{G\left[\frac{2\pi R}{(\Sigma d_i)} t_c + t\right]}$	$E\bar{I}_x$	$\frac{E}{12(1-\nu^2)}\left[t_c^{\,3}\left(\frac{\delta\theta}{\Delta\theta}\right) + t^3\right]$	νD_{22}	$\frac{G}{12}\left[t_c^{\,3}\left(\frac{\delta\theta}{\Delta\theta}\right) + t^3\right]$	$\bar{z}_x$ (Positive for internally stiffened; Negative for externally stiffened)	$-\nu\bar{z}_x$
C		$\left[\frac{(\Sigma d_i)\ t_c}{2\pi R}\right]$	$\frac{1}{E\bar{t}_x}$	$\left(\frac{1}{Et_c}\right)\left(\frac{\Delta_x}{\delta_x}\right)$	0	$\frac{1}{G\left[\frac{2\pi R}{(\Sigma d_i)} t_c\right]}$	$E\bar{I}_x$	$\left[\frac{Et_c^{\,3}}{12(1-\nu^2)}\right]\left(\frac{\delta\theta}{\Delta\theta}\right)$	νD_{22}	$\left[\frac{Gt_c^{\,3}}{12}\right]\left(\frac{\delta\theta}{\Delta\theta}\right)$	0	0

Notes for TABLE X

(a) For convenience, all of the figures shown here depict only externally stiffened configurations. All of the formulas for $\bar{t}_x$, the A_{ij}'s, and the D_{ij}'s apply equally well to internally stiffened configurations. The formulas for the C_{ij}'s include the eccentricity value $\bar{z}_x$ whose sign depends upon the stringer location ($\bar{z}_x$ is positive for internally stiffened configurations and negative for externally stiffened configurations).

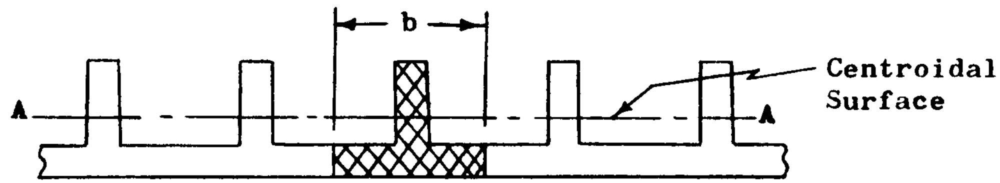
(b) The quantity $\bar{t}_x$ is the wall thickness for a monocoque circular cylinder of the same radius as the middle surface of the stiffened-cylinder basic skin, and of the same total cross-sectional area as the actual composite stiffened wall, including both skin and stringers. The cross section referred to here is obtained by passing a plane through the entire cylinder, normal to the axis of revolution.

(c) The quantity $\bar{I}_x$ is the local longitudinal centroidal running moment of inertia for the flat configuration obtained by unfolding the entire composite circular shell wall. For example, consider the case of a cylinder having a local wall cross section of the type



In such a case, one must consider the unfolded geometry shown below.

<u>Notes for TABLE X (Continued)</u>



After computing I_{A-A} for the cross-hatched area shown, $\overline{I}_x$ is found as follows:

$$\overline{I}_x = \frac{I_{A-A}}{b}$$

(d) The term $(1-\nu^2)$ has been omitted from the formulas for D_{11} since the specified configurations usually provide incomplete restraint to anticlastic bending (see GLOSSARY, Volume I [1]). However, the $(1-\nu^2)$ factor has been retained in all of the formulas for D_{22} since the usually broad axial extent of the skin panel affords restraint to anticlastic bending in the same manner as that customarily recognized for flat plates.

(e) The quantity D_{33} is based on the conservative assumption that the stringers furnish no resistance to twisting deformations.

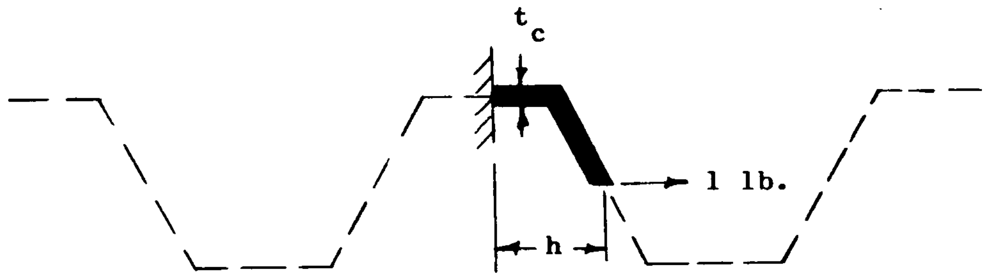
(f) The quantity A_s is the cross-sectional area of a single stringer, and does not include any of the basic cylindrical skin.

(g) The symbol (Σd_i) is used to denote the total peripheral length of the corrugation center-line for the wave-type cross section obtained by passing a plane through the entire cylinder, normal to the axis of revolution. Hence, $(\Sigma d_i) > 2\pi R$ and the total area for the stated corrugation cross section may be taken equal to $(\Sigma d_i)(t_c)$.

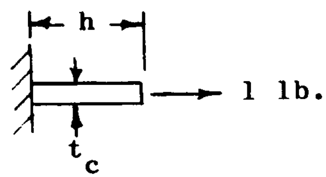
Notes for TABLE X (Continued)

(h) The factor (Δ_x/δ_x) accounts for the accordion-like hoop extensional flexibility of a corrugation. For example, consider a corrugation of the type: 

In this case, the quantity Δ_x is the linear deflection, in the direction of loading, for the point of load application, in the following situation:

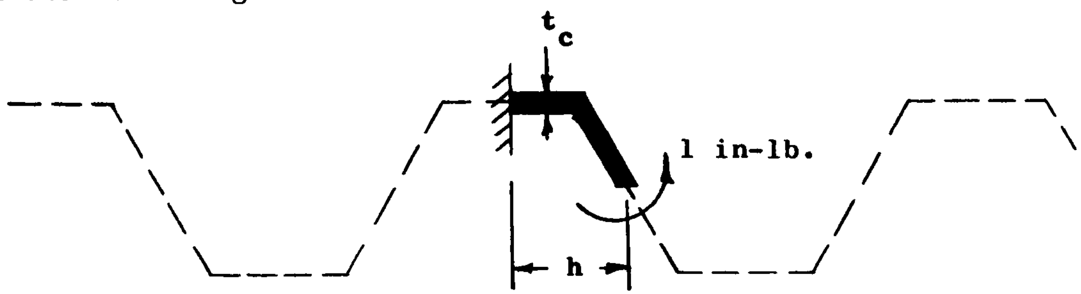


The quantity δ_x is the linear deflection, in the direction of loading, for the point of load application, in the following situation:

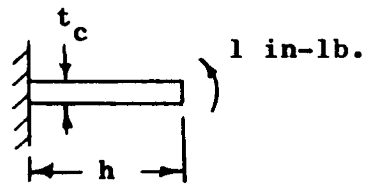


Notes for TABLE X (Continued)

(i) The factor $\left(\frac{\delta\theta}{\Delta\theta}\right)$ accounts for the increased length over which circumferential bending occurs in the case of a corrugation. For example, consider a corrugation of the same type as in note (h) above. In this case, the quantity $\Delta\theta$ is the rotation, in the direction of loading, for the point of load application, in the following situation:



The quantity $\delta\theta$ is the rotation, in the direction of loading, for the point of load application, in the following situation:



Hence it follows that $\left(\frac{\delta\theta}{\Delta\theta}\right) = \frac{2\pi R}{(\Sigma d_i)}$.

SECTION 7

DIGITAL COMPUTER PROGRAMS

7.1 CRITICAL STRESS

This section presents the essential features of General Dynamics Convair digital computer program numbered 4196. This program was developed for the analysis of instability in axially compressed circular cylinders having eccentric stringers but no intermediate rings. To make proper use of the output from the program, one should refer to the instructions furnished in SECTION 4, "ANALYSIS METHOD". The solution is based upon the theoretical and empirical considerations presented in SECTION 2. In particular, the program employs equations (2-24) and (2-25). The output can be obtained in the form of automatically plotted buckling curves or as single-point solutions, as desired. All of the curves presented in SECTION 5.1 and APPENDIX A were obtained by using the automatic plotting option of the program. The input format is shown in Figure 8. Symbols are listed in Table XI. A detailed, card-by-card description of the input follows below. Runs may be stacked.

CARD TYPE 1: One card per run.

Enter PROBLEM IDENTIFICATION in columns 1-72. Alphanumeric characters.

CARD TYPE 2: One card per run.

Enter E (Young's modulus, psi) in columns 1-10 (E10.5).

Enter SIGCY (Compressive yield stress, psi) in columns 11-20 (E10.5).

Enter SIGPL (Stress at assumed proportional limit, psi) in columns 21-30 (E10.5). PRESENT PROGRAM LIMITATION REQUIRES THAT THIS VALUE NOT BE LESS THAN 20,000 PSI.

Enter SIGPT7 (Ramberg-Osgood parameter, $\sigma_{.7}$, psi) in columns 31-40 (E10.5).

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PROGRAM BUCKLING ØF LØNGITUDINALLY STIFFENED CYLINDERS

PROGRAMMER L.S. FØSSUM

WORK ORDER

EXT

DATE

WAP

CARD TYPE:

	1–5	6–10	11–20	21–30	31–40	41–45	46–50	51–55	56–60	61–70	71–80
1	PRØBLEM IDENTIFICATIØN										
2	E		SIGCY	SIGPL	SIGPTY	FNU	VALUE	NCASES	NLRHØX		
3	$LRHØX_1$		$LRHØX_2$	$LRHØX_3$	....	....		....		$LRHØX_{NLRHØX-1}$	$LRHØX_{NLRHØX}$
4	CASE NØ	ØPTIØN	SIGCC	N STAR	RT	LRHØX					

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Figure 8 - Input Format - Program 4196

Enter FNU (Poisson's ratio, ν) in columns 41-45 (F5.3).

Enter VALUEN (Ramberg-Osgood parameter, n) in columns 46-50 (F5.1).

Enter NCASES (number of cases) as right adjusted integer in columns 51-55 (I5).

Enter NLRHOX (number of L'/ρ_x values to be read in on CARD TYPE 3 and used in TABLE and PLOTS option if called for on CARD TYPE 4) as right adjusted integer in columns 56-60 (I5).

CARD TYPE 3: There will be NLRHOX/8 (rounded to higher whole number) cards per run.

Enter LRHOX (slenderness ratio, L'/ρ_x) values, 8 to a card (8E10.5).

For point solutions (NLRHOX=0), omit this card.

CARD TYPE 4: There will be NCASES cards per run.

Enter CASENO (case number) as right adjusted integer in columns 1-5 (I5).

Enter OPTION (TABLE, PLOTS, or POINT) in columns 6-10(A5).

If TABLE, 301 values of R/t are generated evenly spaced on a logarithmic scale and calculations are made for each combination of R/t and L'/ρ_x . If PLOTS, plots are made in addition to calculations of TABLE. If POINT, one set of calculations only is run using the RT and LRHOX values in columns 31-40 and 41-50, respectively, on the same card.

Enter SIGCC (crippling stress, σ_{cc}, psi) in columns 11-20 (E10.5).

Enter NSTAR (minimization factor N*) in columns 21-30 (E10.5).

Enter RT (R/t for use only in POINT option) in columns 31-40 (E10.5).
Not necessary for TABLE or PLOTS options.

Enter LRHOX (L'/ρ_x for use only in POINT option) in columns 41-50 (E10.5). Not necessary for TABLE or PLOTS options.

A sample input coding form is shown in Figure 9.

The program output consists of a listing or a listing plus plots depending upon the option selected. A sample output listing for OPTION = POINT is shown in Figure 10. Typical plots are given in APPENDIX A. A basic flow diagram for the program is presented as Figure 11 and a Fortran listing of the program is shown in Table XII.

TABLE XI - Program 4196 Notation

PROGRAM NOTATION	REPORT NOTATION	DESCRIPTION
C	-	Unity.
CASENO	-	Case number.
CONST1	$\sqrt{3(1-\nu^2)}$	
E	E	Young's Modulus, psi.
ETANCY	$(E_{tan})_{cy}$	Tangent modulus at compressive yield stress, psi.
FNU	ν	Poisson's Ratio.
GAMMAN	N*	Minimization Factor.
ISTOP	-	Indicates R/t value at which no further calculations are made for that L'/ρ_x.
LRHOX	L'/ρ_x	Effective slenderness ratio.
RTBAR	R/t	Radius/thickness ratio.
SIGCY	σ_{cy}	Compressive yield stress, psi.
SIGPL	σ_{PL}	Stress at assumed proportional limit, psi.
SIGPT7	$\sigma_{.7}$	Ramberg-Osgood parameter, psi.
SIGCC	σ_{cc}	Crippling stress, psi.
SIGCR	σ_{cr}	Buckling stress, psi.
SCRCY	$(\sigma_{cr})_{cy}$	Buckling stress using $E_{tan} = (E_{tan})_{cy}$
SWCCY	$(\sigma_{wc})_{cy}$	Wide-column buckling stress using $E_{tan} = (E_{tan})_{cy}$
VALUEN	n	Ramberg-Osgood parameter.
YT	-	Upper limit of plotting grid.

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PROGRAM	PROGRAMMER	WORK ORDER
BUCKLING ØF LØNGITUDINALLY STIFFENED CYLINDERS	L.S. FØSSUM	
	EXT	DATE	WAP

CARD TYPE:

Card type	Line	Columns 1–80
1	1	`              SAMPLE PRØBLEM`
2	2	`10.5   +0638.0   +0320.0   +0337.0   +03  .33 10.0    4`
4	3	`    1PØINT47.5   +03    .2     900.0      85.0`
4	4	`    2PØINT47.5   +03    .8     850.0     115.0`
4	5	`    3PØINT47.5   +03   1.2     800.0     145.0`
4	6	`    4PØINT47.5   +03   1.6     750.0     175.0`

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Figure 9 - Sample Input Data - Program 4196

BUCKLING OF LONGITUDINALLY STIFFENED CYLINDERS

SAMPLE PROBLEM

INITIAL E, PSI	COMPRESSIVE YIELD STRESS,PSI	ELASTIC LIMIT STRESS,PSI	SIGMA PT SEVEN, PSI	POISSONS RATIO	N	NUMBER OF CASES	NO. SLENDERNESS RATIOS
1.0500E 07	3.8000E 04	2.0000E 04	3.7000E 04	0.330	10.0	4	-0

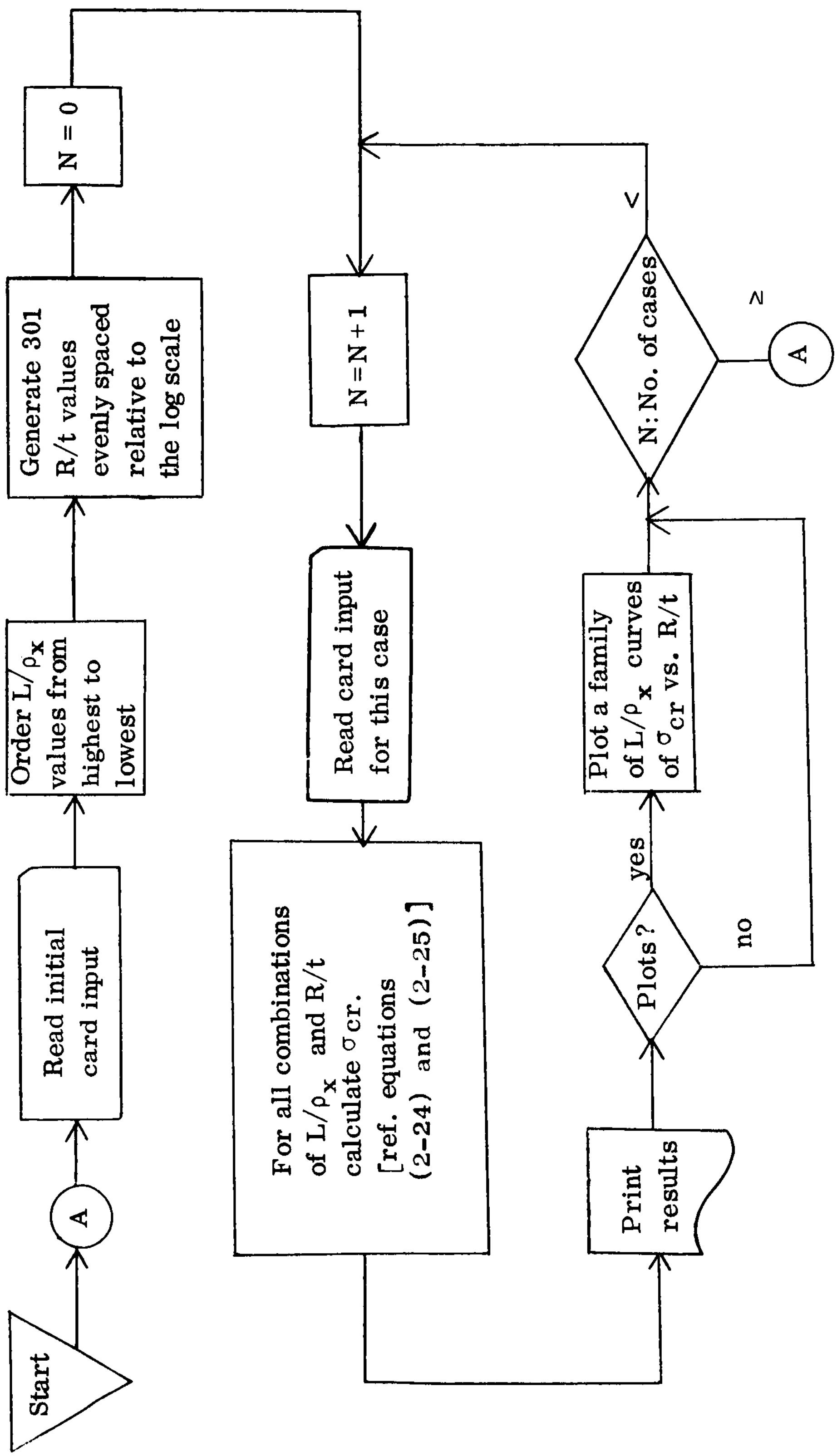
CASE NO.	CRIPPLING STRESS, PSI	N STAR	RADIUS / T	SLENDERNESS RATIO	BUCKLING STRESS, PSI
1	4.750E 04	2.00000E-01	9.000E 02	8.5000E 01	1.5770E 04

CASE NO.	CRIPPLING STRESS, PSI	N STAR	RADIUS / T	SLENDERNESS RATIO	BUCKLING STRESS, PSI
2	4.750E 04	8.00000E-01	8.500E 02	1.1500E 02	1.3880E 04

CASE NO.	CRIPPLING STRESS, PSI	N STAR	RADIUS / T	SLENDERNESS RATIO	BUCKLING STRESS, PSI
3	4.750E 04	1.20000E 00	8.000E 02	1.4500E 02	1.4562E 04

CASE NO.	CRIPPLING STRESS, PSI	N STAR	RADIUS / T	SLENDERNESS RATIO	BUCKLING STRESS, PSI
4	4.750E 04	1.60000E 00	7.500E 02	1.7500E 02	1.7084E 04

Figure 10 - Sample Output Listing - Program 4196



<u>Figure 11 - Flow Diagram - Program 4196</u>

TABLE XII - Fortran Listing - Program 4196

```
$IBFTC MAIN
      COMMON C, CASENO, E, FNU, GAMMAN, ISTOP(25), LRHOX(25), NLAST,
     1        NCASES, NLRHOX, NOP, OPTION, RTBAR(302), SIGCY, SIGPL,
     2        SIGPT7, SIGCC, SIGCR(302,25), SCRCY, VALUEN, YT,
     3         PROBID(12), PI,CONST1
      REAL LRHOX, LRHOXC
      DATA PLOTS, TABLE, POINT /5HPLOTS,5HTABLE,5HPOINT/
C
C     10-10-66  GAMMAN = N STAR
C                RTBAR = RT
C                    C = 1
C
      C=1.0
   50 WRITE (6,51)
   51 FORMAT (1H1,42X,46HBUCKLING OF LONGITUDINALLY STIFFENED CYLINDERS)
   60 READ (5,61) (PROBID(K),K=1,12)
   61 FORMAT (12A6)
   70 WRITE (6,71) (PROBID(K),K=1,12)
   71 FORMAT (//// 29X,12A6 ////)
   80 WRITE (6,81)
   81 FORMAT (7X, 10HINITIAL E, 5X, 11HCOMPRESSIVE 5X, 13HELASTIC LIMIT
    1          5X, 8HSIGMA PT 10X, 8HPOISSONS 21X, 6HNUMBER 3X,
    2          15HNO. SLENDERNESS /10X, 3HPSI 7X, 16HYIELD STRESS,PSI
    3          3X, 10HSTRESS,PSI 6X, 10HSEVEN, PSI 11X, 5HRATIO 13X, 1HN
    4          7X, 8HOF CASES 7X, 6HRATIOS  )
   90 READ (5,91) E, SIGCY, SIGPL, SIGPT7, FNU, VALUEN, NCASES, NLRHOX
   91 FORMAT (4E10.5,F5.3,F5.1,2I5)
  100 WRITE (6,101)E, SIGCY, SIGPL, SIGPT7, FNU, VALUEN, NCASES, NLRHOX
  101 FORMAT (1H0, 1P4E16.4, 0PF16.3, F16.1, I10, I14 )
  105 IF (NLRHOX.EQ.0) GO TO 140
  110 WRITE (6,111)
  111 FORMAT (//// 47X, 37HSLENDERNESS RATIOS FOR AUTOMATIC SEG. // )
  120 READ (5,121) (LRHOX(K),K=1,NLRHOX)
  121 FORMAT ( 8E10.5)
  130 WRITE (6,131)  (LRHOX(K),K=1,NLRHOX)
  131 FORMAT ( 1P6E20.5 )
  140 CONST1 = SQRT ( 3. *(1. - FNU**2 ))
  141 PI=3.14159
  142 CALL SETMIV(125,0,160,176)
  143 CALL SMXYV(1,0)
C
C     CALCULATE UPPER LIMIT OF GRID
      XX=SIGCY+15000.
      YT=XX-AMOD(XX,1000.0.)
  150 CALL SEARCH (LRHOX,NLRHOX)
  151 CALL RTBARC (RTBAR,NRTBAR)
  160 DO 255  NREAD=1,NCASES
C     STORE R/TBAR FOR POINT OPTION IN RTBAR(302) AND L/RX IN LRHOX(25).
  170 READ (5,171) CASENO,OPTION,SIGCC,GAMMAN,RTBAR(302),LRHOX(25)
  171 FORMAT (I5,A5,4F10.5)
  175 NOP=0
```

TABLE XII - Fortran Listing - Program 4196
(Continued)

```
  180 IF(PLOTS.EQ.OPTION) NOP=1
  190 IF(TABLE.EQ.OPTION) NOP=2
  200 IF(POINT.EQ.OPTION) NOP=3
  210 IF(NOP.NE.0) GO TO 250
  220 WRITE (6,221) CASENO
  221 FORMAT (//// 49X, 28HILLEGAL OPTION FOR CASE NO. ,I5)
  230 GO TO 255
  250 CALL COMPUT
  255 CONTINUE
  260 GO TO 50
  270 END
$IBFTC SEARCH
      SUBROUTINE SEARCH (LRHOX,NLRHOX)
      DIMENSION LRHOX(25)
C     TO PLACE INPUT L/RX IN ORDER FROM HIGHEST TO LOWEST.
      REAL LRHOX, LRHOXD
  100 NLRX1=NLRHOX-1
  150 DO 650 K=1,NLRX1
  200 TEMP=LRHOX(K)
  250 K1=K+1
  275 ITRANS=0
  300 DO 500 I=K1,NLRHOX
  350 IF (TEMP.GE.LRHOX(I)) GO TO 500
  375 ITRANS=1
  400 NBLANK=I
  450 TEMP=LRHOX(I)
  500 CONTINUE
  525 IF (ITRANS.EQ.0) GO TO 650
  550 LRHOX(NBLANK)=LRHOX(K)
  600 LRHOX(K)=TEMP
  650 CONTINUE
  700 RETURN
  750 END
$IBFTC RTBARC
      SUBROUTINE RTBARC ( RTBAR, NRTBAR )
C     TO GENERATE 301 VALUES OF RTBAR FROM 10 TO 10,000
C     EVENLY SPACED RELATIVE TO LOG SCALE.
      DIMENSION RTBAR(301)
  100 NRTBAR = 301
  200 RTBAR(1) = 10000.
  300 RTBAR(301) = 10.
  400 DO 800 I = 1,299
  500 I1 = 301-I
  600 EXP = 1.0 + FLOAT(I) * .01
  700 RTBAR(I1) = 10. **EXP
  800 CONTINUE
  900 RETURN
 1000 END
$IBFTC COMPUT
      SUBROUTINE COMPUT
C
```

TABLE XII - Fortran Listing - Program 4196
(Continued)

```
      COMMON C, CASENO, E, FNU, GAMMAN, ISTOP(25), LRHOX(25), NLAST,
     1       NCASES, NLRHOX, NOP, OPTION, RTBAR(302), SIGCY, SIGPL,
     2       SIGPT7, SIGCC, SIGCR(302,25), SCRCY, VALUEN, YT,
     3        PROBID(12), PI,CONST1
      REAL LRHOX, LRHOXD
C
C     THIS SUBROUTINE WILL DO THE FOLLOWING ACCORDING TO THE OPTION
C
C     POINT OPTION
C     THE BUCKLING STRESS VALUE IS COMPUTED FOR THE INPUT L/RX AND R/T.
C
C     TABLE OPTION
C     301 R/TBARS ARE GENERATED FROM 10 TO 10,000 (IN SUB. RTBARC).
C     BUCKLING STRESS VALUES ARE CALCULATED FOR EACH L/RX AND R/TBAR.
C
C     PLOTS OPTION
C     SAME AS TABLE OPTION. IN ADDITION A PLOT OF BUCKLING STRESS VS.
C     R/TBAR IS MADE FOR EACH L/RX VALUE.
C
C     STEP NOS.REFER TO STEPS IN OUTLINE OF PROBLEM WRITTEN
C     BY GEORGE SMITH. SEE DOCUMENTATION.
C
C
  100 GO TO (125,125,135),NOP
C
C     LOOP FOR TABLE AND PLOTS OPTIONS.
C
  125 J1=1
  126 J2=NLRHOX
  127 I1=1
  128 I2=301
  129 GO TO 150
C
C     POINT OPTION. CALCULATE ONLY FOR ONE POINT USING SPECIAL VALUES
C     STORED IN RTBAR(302) AND LRHOX(25).
C
  135 J1=25
  136 J2=25
  137 I1=302
  138 I2=302
C
C     STEPS 1 - 3
  150 LRHOXD = SQRT(2.*C) * PI * SQRT(E/SIGCC)
  160 ETANCY = E * (1./(VALUEN * (SIGCY/SIGPT7)**(VALUEN-1.) + 1.))
  170 DO 2080  J=J1,J2
C
C     STEPS 5 AND 6
  190 IF (LRHOX(J).GE.LRHOXD) GO TO 300
C
C     PROCEED AS IN STEPS 22 - 41 (IGO=2)
  200 IGO = 2
```

TABLE XII - Fortran Listing - Program 4196
(Continued)

```
C
C     STEPS 22, 23, AND 24
  210 STEP22 = (C* PI**2 * ETANCY)/(LRHOX(J)**2)
  220 STEP23 = SIGCC - ( (SIGCC**2 * LRHOX(J)**2)/(4.*C*PI**2 * E) )
  230 SWCCY = AMIN1(STEP22,STEP23)
  240 AA = SWCCY
  250 GO TO 325
C
C     PROCEED AS IN STEPS 7 - 21 (IGO=1)
  300 IGO = 1
  310 AA = (C * PI**2 * ETANCY) / (LRHOX(J)**2)
C
C     STEP 4
  325 DO 2060  I=I1,I2
C
C     STEPS (7 AND 8) AND (25 AND 26)
  350 SCRCY = AA + GAMMAN * (ETANCY/CONST1) * (1./RTBAR(I))
  360 IF (SCRCY - SIGCY) 450,370,400
C
C     STEPS 10 AND 28
  370 SIGCR(I,J) = SCRCY
      IF (SIGCR(I,J).LE.SIGCC) GO TO 2060
      ISTOP(J)=I
      GO TO 2080
C
C     STEPS 9 AND 27
  400 IF (I.EQ.302) GO TO 425
  420 ISTOP(J)=I
  421 GO TO 2080
  425 ISTOP(J)=303
  430 GO TO 2080
C
C     STEPS (11 - 16) AND (29 - 34)
  450 GO TO (460,500),IGO
  460 BB = (C * PI**2 * E) / LRHOX(J)**2
  470 GO TO 550
  500 BB = SIGCC - (SIGCC**2 * LRHOX(J)**2) / (4. * C * PI**2 * E)
  550 SCRBB = BB + GAMMAN * (E / CONST1) * (1./RTBAR(I))
  560 IF (SCRBB.GT.SIGPL) GO TO 600
  570 SIGCR(I,J) = SCRBB
      IF (SIGCR(I,J).LE.SIGCC) GO TO 2060
      ISTOP(J)=I
      GO TO 2080
  600 IF (SCRBB.GE.SIGCY) GO TO 660
  620 SCRNEW = SCRBB
  640 GO TO 680
  660 SCRNEW = SIGCY
C
C     STEPS (17 - 21) AND (35 -41)
C
C     INITIALIZE
```

TABLE XII - Fortran Listing - Program 4196
(Continued)

```
  680 X = 12500.
  700 SCRNEW = SCRNEW + X
  720 NCNT = 0
C
C     ENTER SCHEME TO CLOSE IN ON OUTPUT VALUE
  740 NCNT =NCNT + 1
  760 SCRNEW = SCRNEW - X
  780 ETANNU = E*(1./(VALUEN * (SCRNEW/SIGPT7)**(VALUEN-1.) +1.))
  800 GO TO (920,820),IGO
  820 STEP36 = (C * PI**2 * ETANNU) / LRHOX(J)**2
  840 STEP37 = SIGCC - (SIGCC**2 * LRHOX(J)**2) / (4. * C * PI**2 * E)
  860 SWC38 = AMIN1(STEP36,STEP37)
  880 CC = SWC38
  900 GO TO 940
  920 CC = (C * PI**2 * ETANNU) / LRHOX(J)**2
C
C     STEPS 18 AND 39 (EFN 940)
  940 SCRNU = CC + GAMMAN * (ETANNU/CONST1) * (1./RTBAR(I))
  960 IF (SCRNU.LT.SCRNEW) GO TO 740
 1000 IF (NCNT.EQ.1) GO TO 1020
 1010 IF ( X.GT.100.) GO TO 1080
 1020 SIGCR(I,J) = SCRNEW
      IF (SIGCR(I,J).LE.SIGCC) GO TO 2060
      ISTOP(J)=I
      GO TO 2080
 1080 SCRNEW = SCRNEW + X
 2010 X = X/5.
 2020 GO TO 760
 2060 CONTINUE
 2070 ISTOP(J)=302
 2080 CONTINUE
 3000 CALL OUTPUT (J1, J2, I1, I2)
 3020 GO TO (4000,5000,5000),NOP
 4000 CALL SIGPLT
 5000 RETURN
 6000 END
$IBFTC OUTPUT
      SUBROUTINE OUTPUT(J1, J2, I1, I2)
      COMMON C, CASENO, E, FNU, GAMMAN, ISTOP(25), LRHOX(25), NLAST,
     1        NCASES, NLRHOX, NOP, OPTION, RTBAR(302), SIGCY, SIGPL,
     2        SIGPT7, SIGCC, SIGCR(302,25), SCRCY, VALUEN, YT,
     3         PROBID(12), PI,CONST1
      REAL LRHOX, LRHOXL

      SUBROUTINE TO PRINT OUT OUTPUT.

  100 WRITE (6,101)
  101 FORMAT ( //// 20X,9HCRIPPLING 12X, 6HFIXITY 9X, 10HCORRECTION 28X,
     1        11HSLENDERNESS 9X, 8HBUCKLING  // 5X, 8HCASE NO. 6X,
     2        11HSTRESS, PSI 11X, 6HFACTOR 11X, 6HFACTOR 11X,
     3        12HRADIUS/T BAR 10X, 5HRATIO 11X, 11HSTRESS, PSI ///)
```

TABLE XII - Fortran Listing - Program 4196
(Continued)

```
  150 IF (ISTOP(J1).EQ.1) GO TO 200
  175 IF (ISTOP(J1).NE.303) GO TO 300
  200 WRITE (6,201) CASENO,SIGCC,C,GAMMAN,LRHOX(J1)
  201 FORMAT (1H I9,1P2E19.3,1PE19.5,19X,
     *         27HNO CALCULATIONS FOR L/RX = ,1PE12.4)
  250 GO TO (500,500, 900),NOP
  300 WRITE (6,301) CASENO,SIGCC,C,GAMMAN,RTBAR(I1),LRHOX(J1),
     *              SIGCR(I1,J1)
  301 FORMAT (1H ,I9,1P2E19.3,1PE19.5,1PE19.3,1P2E19.4)
      IF (NOP.EQ.3) GO TO 900
  399 II=ISTOP(1)-1
  400 WRITE (6,401)  (RTBAR(I),LRHOX(1), SIGCR(I,1), I=2,II)
  401 FORMAT (67X, 1PE19.3, 1P2E19.4)
  425 WRITE (6,426) LRHOX(1)
  426 FORMAT (1H ,74X,34HNO FURTHER CALCULATIONS FOR L/RX= ,1PE12.4)
  500 DO 800  J=2,J2
  550 IF (ISTOP(J).NE.1) GO TO 675
  600 WRITE (6,601) LRHOX(J)
  601 FORMAT(///  75X, 27HNO CALCULATIONS FOR L/RX = ,1PE12.4)
  650 GO TO 800
  675 II=ISTOP(J)-1
  700 WRITE (6,701) (RTBAR(I),LRHOX(J),SIGCR(I,J),I=1,II)
  701 FORMAT (///(66X,1PE19.3,1P2E19.4))
  750 WRITE (6,751) LRHOX(J)
  751 FORMAT (75X,34HNO FURTHER CALCULATIONS FOR L/RX= ,1PE12.4)
  800 CONTINUE
  900 RETURN
 1000 END
$IBFTC OUTPUT
      SUBROUTINE OUTPUT(J1, J2, I1, I2)
      COMMON C, CASENO, E, FNU, GAMMAN, ISTOP(25), LRHOX(25), NLAST,
     1       NCASES, NLRHOX, NOP, OPTION, RTBAR(302), SIGCY, SIGPL,
     2       SIGPT7, SIGCC, SIGCR(302,25), SCRCY, VALUEN, YT,
     3        PROBID(12), PI,CONST1
      REAL LRHOX, LRHOXD
C
C     SUBROUTINE TO PRINT OUT OUTPUT.
C
  100 WRITE (6,101)
  101 FORMAT (//// 26X,9HCRIPPLING 52X,11HSLENDERNESS 12X,8HBUCKLING //
     1         9X,8HCASE NO. 8X,11HSTRESS, PSI 11X,6HN STAR 14X,
     2         10HRADIUS / T 13X,5HRATIO 13X,11HSTRESS, PSI ///)
  150 IF (ISTOP(J1).EQ.1) GO TO 200
  175 IF (ISTOP(J1).NE.303) GO TO 300
  200 WRITE (6,201) CASENO,SIGCC,GAMMAN,LRHOX(J1)
  201 FORMAT (1H ,I13,1PE21.3,E21.5,24X,
     *         27HNO CALCULATIONS FOR L/RX = ,1PE12.4)
  250 GO TO (500,500, 900),NOP
  300 WRITE (6,301) CASENO,SIGCC,GAMMAN,RTBAR(I1),LRHOX(J1),
     *              SIGCR(I1,J1)
  301 FORMAT (1H ,I13,1PE21.3,E21.5,E21,3,2E21.4)
```

TABLE XII - Fortran Listing - Program 4196
(Continued)

```
      IF (NOP.EQ.3) GO TO 900
  399 II=ISTOP(1)-1
  400 WRITE (6,401)  (RTBAR(I),LRHOX(1), SIGCR(I,1), I=2,II)
  401 FORMAT (56X,1PE21.3,2E21.4)
  425 WRITE (6,426) LRHOX(1)
  426 FORMAT (1H ,70X,34HNO FURTHER CALCULATIONS FOR L/RX= ,1PE12.4)
  475 IF(NLRHOX.EQ.1) GO TO 900
  500 DO 800  J=2,J2
  550 IF (ISTOP(J).NE.1) GO TO 675
  600 WRITE (6,601) LRHOX(J)
  601 FORMAT(///  80X, 27HNO CALCULATIONS FOR L/RX = ,1PE12.4)
  650 GO TO 800
  675 II=ISTOP(J)-1
  700 WRITE (6,701) (RTBAR(I),LRHOX(J),SIGCR(I,J),I=1,II)
  701 FORMAT (///(56X,1PE21.3,2E21.4))
  750 WRITE (6,751) LRHOX(J)
  751 FORMAT (71X,34HNO FURTHER CALCULATIONS FOR L/RX= ,1PE12.4)
  800 CONTINUE
  900 RETURN
 1000 END
$IBFTC SIGPLT
      SUBROUTINE SIGPLT
      COMMON C, CASENO, E, FNU, GAMMAN, ISTOP(25), LRHOX(25), NLAST,
     1         NCASES, NLRHOX, NOP, OPTION, RTBAR(302), SIGCY, SIGPL,
     2         SIGPT7, SIGCC, SIGCR(302,25), SCRCY, VALUEN, YT,
     3          PROBID(12), PI,CONST1
      REAL LRHOX, LRHOXD
C     USED FOR PLOTS OPTION. WILL PLOT SIGCR VS R/TBAR FOR VARIOUS L/RX.
C
C     SET GRID
  100 CALL GRID1V (4,10.,10000.,0.,YT,1.,2000.,0, 5,-10,- 5,6,6)
C
C     PRINT SIGCC AND N STAR AT TOP
  150 CALLPRINTV (-19,19HCRIPPLING STRESS = ,192,876)
  151 CALL LABLV (SIGCC,344,876,-4,1,1)
  170 CALL PRINTV(-9,9HN STAR = ,872,876)
  171 CALL LABLV (GAMMAN,944,876,7,1,3)
C
C     PRINT SIGCR TITLE DOWN SIDE
C
  180 CALL APRNTV(0,-14,-19,19HBUCKLING STRESS PSI,76,637)
C     PRINT RADUIS / T AT BOTTOM
  190 CALL PRINTV(-10,10HRADIUS / T,551,120)
C
C     PRINT MAIN TITLE AT BOTTOM
  200 CALL RITE2V(328,77,1023,90,1,31,-1,31HCOMPRESSIVE BUCKLING STRESS
     *FOR,NLAST)
  201 CALL RITE2V(301,45,1023,90,1,34,-1,34HLONGITUDINALLY STIFFENED CYL
     *INDERS,NLAST)
  202 CALL RITE2V(364,13,1023,90,1,10,-1,10HMATERIAL -,NLAST)
```

TABLE XII - Fortran Listing - Program 4196
(Continued)

```
C     PLOT CURVES
  300 DO 700  J=1,NLRHOX
  310 IF (ISTOP(J).EQ.1) GO TO 700
  320 RTB = RTBAR(1)
  330 SIG = SIGCR(1,J)
  340 IX1 = NXV(RTB)
  350 IY1 = NYV(SIG)
      II=ISTOP(J)-1
  400 DO 600  I=2,II
  410 RTB = RTBAR(I)
  420 SIG = SIGCR(I,J)
  440 IX2 = NXV(RTB)
  450 IY2 = NYV(SIG)
  500 DO 501 NN=1,3
  501 CALL LINEV(IX1,IY1,IX2,IY2)
  550 IX1 = IX2
  560 IY1 = IY2
  600 CONTINUE
  700 CONTINUE
C
C     PLOT CUT OFF LINE AT SIGMA CC FROM X=10 TO X OF LAST POINT PLOTTED
      IF (SIGCC.GT.YT) GO TO 900
  750 NSCC=NYV(SIGCC)
  775 NTEN=NXV(10.)
  800 DO 801 MM=1,3
  801 CALL LINEV (NTEN, NSCC, IX1, NSCC)
  900 RETURN
 1000 END
```

7.2 MINIMIZATION FACTOR N*

This section presents the essential features of General Dynamics Convair digital computer program numbered 4235. This program was developed to establish appropriate values for the minimization factor N* used in the analysis of instability in axially compressed circular cylinders having eccentric stringers but no intermediate rings. To make proper use of the output from the program, one should refer to the instructions furnished in SECTION 4, "ANALYSIS METHOD". The programmed computations are based upon the theoretical considerations presented in SECTION 2. In particular, equation (2-22) is employed. The output can be obtained in the form of automatically plotted data or as single-point solutions, as desired. All of the curves presented in SECTION 5.2 were generated with the assistance of the automatic plotting option of the program. The input format is shown in Figure 12. Symbols are listed in Table XIII. A detailed, card-by-card description of the input follows below. Runs may be stacked.

CARD TYPE 1: One card per run.

Enter PROBLEM IDENTIFICATION anywhere in columns 1-60.

Alphanumeric characters.

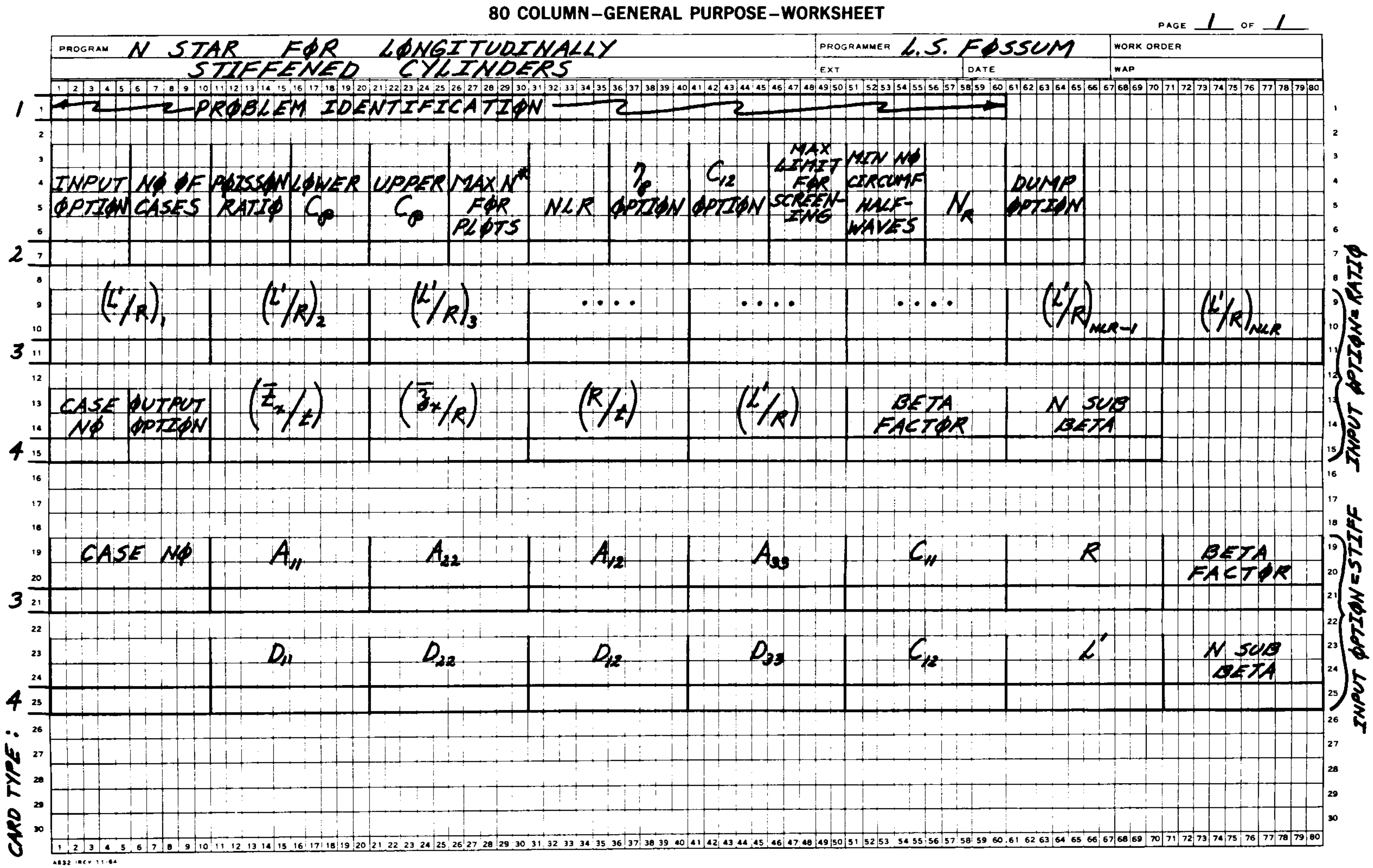
CARD TYPE 2: One card per run.

Enter INPUT OPTION (RATIO or STIFF) in columns 1-5.
This option permits the user to choose between alternative formats for cards 3 and 4.

Enter NO OF CASES as right adjusted integer in columns 6-10 (I5).

Enter POISSON'S RATIO (ν) in columns 11-15 (F5).

Enter LOWER C_β in columns 16-20 (F5). Output is listed for critical $\beta(=\beta^*)$ and also for $\beta=(\text{LOWER } C_\beta) * \beta^*$.
The value (LOWER C_β) = 0.99 will usually be suitable.



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PROGRAM: N STAR FØR LØNGITUDINALLY STIFFENED CYLINDERS

PROGRAMMER: L.S. FØSSUM

WORK ORDER / EXT / DATE / WAP

CARD TYPE:

Card type 1:

PRØBLEM IDENTIFICATIØN

Card type 2:

INPUT ØPTIØN	NØ ØF CASES	PØISSØN RATIØ	LØWER C_ϕ	UPPER C_ϕ	MAX N^* FØR PLØTS	NLR	η_ϕ ØPTIØN	C_{12} ØPTIØN	MAX LIMIT FØR SCREEN-ING	MIN NØ CIRCUMF HALF-WAVES	N_R	DUMP ØPTIØN

INPUT ØPTIØN=RATIØ

Card type 3:

$(L'/R)_1$	$(L'/R)_2$	$(L'/R)_3$	....	....	....	$(L'/R)_{NLR-1}$	$(L'/R)_{NLR}$

Card type 4:

CASE NØ	ØUTPUT ØPTIØN	$(\bar{t}_x/t)$	$(\bar{z}_x/R)$	(R/t)	(L'/R)	BETA FACTØR	N SUB BETA

INPUT ØPTIØN=STIFF

Card type 3:

CASE NØ	A_{11}	A_{22}	A_{12}	A_{33}	C_{11}	R	BETA FACTØR

Card type 4:

	D_{11}	D_{22}	D_{12}	D_{33}	C_{12}	L'	N SUB BETA

Figure 12 - Input Format - Program 4235

Enter UPPER C_β in columns 21-25 (F5). Output is listed for critical β ($=\beta^*$) and also for β = (UPPER C_β) $\times \beta^*$. The value (UPPER C_β) = 1.01 will usually be suitable.

Enter MAX N* FOR PLOTS in columns 26-30 (F5). This sets the value for the uppermost grid line in output plots. Any of the following values may be inserted here:

0.5	0.9	4.0	50.0
0.6	1.0	5.0	70.0
0.7	2.0	6.0	
0.8	3.0	10.0	

This entry is left blank when no plots are to be made.

Enter NLR (the number of L/R ratios to be included in plots and/or tables that result from automatic sequencing operations) as right adjusted integer in columns 31-35 (I5). Will be left blank when only point solutions are to be obtained.

Enter η_p OPTION as right adjusted integer in columns 36-40 (I5). Always insert the number 1 here except when it is desired to eliminate the η_p contribution. In the latter case, leave columns 36-40 blank.

Enter C_{12} OPTION as right adjusted integer in columns 41-45 (I5). Always insert the number 1 here except when it is desired to eliminate the C_{12} contribution. In the latter case, leave columns 41-45 blank.

Enter MAX LIMIT FOR SCREENING in columns 46-50 (F5). This is a cut-off value used in the minimization process and must be set

(a) greater than the output N* value for cases where INPUT OPTION = RATIO

and

(b) greater than the output MINIMUM VALUE for cases where INPUT OPTION = STIFF.

When these conditions are not satisfied, the printed results cannot be believed. In such instances, one should increase the input MAX LIMIT FOR SCREENING and rerun the program. Except for externally stiffened configurations, the value (MAX LIMIT FOR SCREENING) = 1.0 will usually be suitable. For externally stiffened cylinders, a value of 70.0 will usually be satisfactory.

Enter MIN NO CIRCUMF HALF-WAVES in columns 51-55 (F5). This is the minimum number of circumferential half-waves considered to be permissible for non-axisymmetric buckle patterns. A value of 2.0 should usually be inserted here.

Enter N_R as a right adjusted integer in columns 56-60 (I5). This input constitutes the number of refinement cycles used to improve the accuracy of final computed values. Whenever (BETA FACTOR) $\leq$ 1.05, the value $N_R = 5$ should usually be satisfactory. Each single refinement cycle essentially cuts the final β screening increment in half.

Enter DUMP OPTION as a right adjusted integer in columns 61-65 (I5). Insert the number 1 whenever supplementary diagnostic output data is to be printed out. Otherwise, leave blank.

CARD TYPE 3 (RATIO OPTION):

There will be NLR/8 (rounded to higher whole number) cards per run.

Enter (L′/R) values, 8 to a card (8E10.5).

When NLR = 0, omit this card.

CARD TYPE 4 (RATIO OPTION):

There will be NO OF CASES cards per run.

Enter CASE NO as right adjusted integer in columns 1-5 (I5).

Enter OUTPUT OPTION as right adjusted integer in columns 6-10 (I5).

1 = Tables with no plots
2 = Tables plus plots
3 = Plots with no tables
4 = Point solution

Enter the thickness ratio ($\bar{t}_x/t$) in columns 11-20 (E10.5).

Enter the eccentricity-to-radius ratio ($\bar{z}_x/R$) in columns 21-30 (E10.5). Positive for internal stiffening; negative for external stiffening.

For point solutions (OUTPUT OPTION = 4) only, enter the radius-to-skin thickness ratio (R/t) in columns 31-40 (E10.5).

For point solutions (OUTPUT OPTION = 4) only, enter the effective length-to-radius ratio (L′/R) in columns 41-50 (E10.5).

Enter BETA FACTOR in columns 51-60 (E10.5). This is a stepping factor used in the minimization process. That is, screening is performed involving β values computed from

$$\beta_{i+1} = (\beta_i) * (\text{BETA FACTOR}) \tag{7-1}$$

The value (BETA FACTOR) = 1.02 should be suitable for most applications.

Enter N SUB BETA (N_β) as right adjusted integer in columns 61-70 (I10). This is a cut-off value used in the minimization process. For further clarification, see reference 8. It is recommended that the selected value for N_β lie within the following limits:

$$50 < N_\beta < 300 \tag{7-2}$$

The program has built-in safeguards which insure that the particular value selected here will not, in any way, influence the accuracy of the computations. Only the machine time will be affected. The value $N_\beta = 150$ will be suitable for most applications.

CARD TYPE 3 (STIFF OPTION):

There will be one of these cards for each case.

Enter CASE NO as right adjusted integer in columns 1-10 (I 10).

Enter the elastic constant A_{11} in columns 11-20 (E10.5).

Enter the elastic constant A_{22} in columns 21-30 (E10.5).

Enter the elastic constant A_{12} in columns 31-40 (E10.5).

Enter the elastic constant A_{33} in columns 41-50 (E10.5).

Enter the eccentricity coupling constant C_{11} in columns 51-60 (E10.5).

Enter R (radius to the middle surface of the basic cylindrical skin) in columns 61-70 (E10.5).

Enter BETA FACTOR in columns 71-80 (E10.5). This is a stepping factor used in the minimization process. That is, screening is performed involving β values computed from

$$\beta_{i+1} = (\beta_i)*(\text{BETA FACTOR}) \qquad (7\text{-}3)$$

The value (BETA FACTOR) = 1.02 should be suitable for most applications.

CARD TYPE 4 (STIFF OPTION):

There will be one of these cards for each case.

Enter the elastic constant D_{11} in columns 11-20 (E10.5).

Enter the elastic constant D_{22} in columns 21-30 (E10.5).

Enter the elastic constant D_{12} in columns 31-40 (E10.5).

Enter the elastic constant D_{33} in columns 41-50 (E10.5).

Enter the eccentricity coupling constant C_{12} in columns 51-60 (E10.5).

Enter the effective length L' (= L/m) in columns 61-70 (E10.5).

Enter N SUB BETA (N_β) as right adjusted integer in columns 71-80 (I10). This is a cut-off value used in the minimization process. For further clarification, see reference 8. It is recommended that the selected value for N_β lie within the following limits:

$$50 < N_\beta < 300 \qquad (7\text{-}4)$$

The program has built-in safeguards which insure that the particular value selected here will not, in any way, influence the accuracy of the computations. Only the machine time will be affected. The value N_β = 150 will be suitable for most applications.

A sample input coding form is shown in Figure 13.

The program output consists of a listing and/or plots depending upon the options selected. A sample output listing for

INPUT OPTION = RATIO

OUTPUT OPTION = 1

is shown in Figure 14. Typical plots are given in SECTION 5.2. A basic flow diagram for the program is presented in Figure 15 and a Fortran listing of the program is shown in Table XIV.

TABLE XIII - Program 4235 Notation

PROGRAM NOTATION	REPORT NOTATION	DESCRIPTION
ALPHA	α	Parameter defined in equations (2-3).
AXISYM	N AXISYM	See reference 8.
A11	A_{11}	Elastic constant.
A12	A_{12}	Elastic constant.
A22	A_{22}	Elastic constant.
A33	A_{33}	Elastic constant.
BASIS	---	Basis (see reference 8).
BETAO	---	Initial β .
BSTAR	β^*	Critical β .
CASENO	---	Case number.
CBETAL	Lower C_β	See description of input for CARD TYPE 2.
CBETAU	Upper C_β	See description of input for CARD TYPE 2.
C11	C_{11}	Eccentricity coupling constant.
C12	C_{12}	Eccentricity coupling constant.
C12OP	C_{12} Option	See description of input for CARD TYPE 2.
DBETA	---	Increment ($\Delta\beta$) in β .
D11	D_{11}	Elastic constant.
D12	D_{12}	Elastic constant.
D22	D_{22}	Elastic constant.
D33	D_{33}	Elastic constant.
ETAPOP	η_p Option	See description of input for CARD TYPE 2.
KINPUT	Input Option	STIFF or RATIO.

TABLE XIII - Program 4235 Notation (Cont'd.)

PROGRAM NOTATION	REPORT NOTATION	DESCRIPTION
KOUTPT	Output Option	1 = Tables only 2 = Tables plus plots 3 = Plots only 4 = Point solution
LEFTN	Left N	See reference 8.
LPRIME	L'	Effective length $\left(= \frac{L}{m}\right)$
LR	(L'/R)	Effective Length/Radius
MSTAR	Critical Lower Case N	Critical value for n (the number of circumferential full waves).
NCASES	---	Number of cases.
NLR	---	Number of (L'/R) ratios for automatic sequencing.
NRT	---	Number of (R/t) values for tables and/or plots.
NSTAR	N^*	Minimization factor.
NU	ν	Poisson's ratio.
PLTMAX	---	Max. N^* for plots.
R	R	Radius.
RIGHTN	Right N	See reference 8.
RT	(R/t)	Radius/Skin Thickness.
SINSEQ	---	Sign sequence (see reference 8).
TBART	$(\bar{t}_x/t)$	Thickness ratio.
ZBARR	$(\bar{z}_x/R)$	(Eccentricity/Radius) Ratio

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PROGRAM	PROGRAMMER	WORK ORDER
N STAR FØR LØNGITUDINALLY STIFFENED CYLINDERS	EXT	DATE	WAP

CARD TYPE:

Card type	Line	Entries (card columns)
1	1	SAMPLE PRØBLEM (col. 10)
2	2	RATIØ (cols. 1–5); 1 (col. 10); .3 (col. 14); .99 (col. 20); 1.01 (cols. 22–25); 1.0 (cols. 28–30); 2 (col. 35); 1 (col. 40); 1 (col. 45); 5.0 (cols. 48–50); 2.0 (cols. 53–55); 5 (col. 60)
3	3	.1 (col. 5); .12 (cols. 15–17)
4	4	1 (col. 5); 1 (col. 10); 1.2 (cols. 14–16); .03 (cols. 25–27); 1.02 (cols. 54–57); 150 (cols. 68–70)

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Figure 13 - Sample Input Data - Program 4235

N STAR FOR LONGITUDINALLY STIFFENED CYLINDERS   PROGRAM 4235

SAMPLE PROBLEM

INPUT OPTION RATIO	NUMBER OF CASES	POISSONS RATIO	LOWER C SUB BETA	UPPER C SUB BETA	MAX N STAR FOR PLOTS	NLR	ETA SUB P OPTION	C12 OPTION	N SUB R	SCREENING LIMIT	MIN NO CIRC HALF WAVES
	1	0.300	0.990	1.010	1.0	2	1	1	5	5.000	2.00

L PRIME/R RATIOS FOR AUTOMATIC SEQUENCING

1.00000E-01   1.20000E-01

CASE NO.	OUTPUT OPTION	T BAR/T	Z BAR/R	BETA FACTOR	N SUB BETA
1	1	1.20000E 00	3.00000E-02	1.02000E 00	150

L PRIME/R
1.00000E-01

R/T	BETA STAR	N STAR	LEFT N	RIGHT N	CRITICAL LOWER CASE N	N AXISYM	SIGN SEQUENCE	BASIS
1.00000E 02	1.87656E 00	9.31000E-01	9.34069E-01	9.33869E-01	1.75220E 01	1.36253E 01	4	2
1.60941E 02	1.83749E 00	5.92820E-01	5.97492E-01	5.97841E-01	1.78945E 01	2.19287E 01	4	2
2.63027E 02	1.82164E 00	3.66345E-01	3.74045E-01	3.74569E-01	1.80502E 01	3.58382E 01	4	2
4.29867E 02	1.81489E 00	2.25001E-01	2.38775E-01	2.37346E-01	1.81174E 01	5.85707E 01	4	2
7.02534E 02	1.81264E 00	1.37872E-01	1.60425E-01	1.58035E-01	1.81398E 01	9.57224E 01	4	2
1.14815E 03	1.81264E 00	8.44293E-02	1.18011E-01	1.20547E-01	1.81398E 01	1.56440E 02	4	2
1.87644E 03	1.81152E 00	5.16691E-02	1.11742E-01	1.05721E-01	1.81511E 01	2.55670E 02	4	2
3.06667E 03	1.81152E 00	3.16174E-02	1.28566E-01	1.21141E-01	1.81511E 01	4.17844E 02	4	2
5.01188E 03	1.81152E 00	1.93495E-02	1.77042E-01	1.66384E-01	1.81511E 01	6.82885E 02	4	2

L PRIME/R
1.20000E-01

R/T	BETA STAR	N STAR	LEFT N	RIGHT N	CRITICAL LOWER CASE N	N AXISYM	SIGN SEQUENCE	BASIS
1.00000E 02	1.83522E 00	6.79806E-01	6.82119E-01	6.81915E-01	1.49306E 01	1.03233E 01	4	2
1.60941E 02	1.79701E 00	4.32789E-01	4.36507E-01	4.36276E-01	1.52481E 01	1.66145E 01	4	2
2.63027E 02	1.78150E 00	2.67433E-01	2.73674E-01	2.73023E-01	1.53807E 01	2.71531E 01	4	2
4.29867E 02	1.77600E 00	1.64246E-01	1.74176E-01	1.73671E-01	1.54284E 01	4.43765E 01	4	2
7.02534E 02	1.77380E 00	1.00640E-01	1.16939E-01	1.15991E-01	1.54475E 01	7.25248E 01	4	2
1.14815E 03	1.77271E 00	6.16234E-02	8.91204E-02	8.58953E-02	1.54571E 01	1.18528E 02	4	2
1.87644E 03	1.77271E 00	3.77071E-02	8.11726E-02	7.87970E-02	1.54571E 01	1.93711E 02	4	2
3.06667E 03	1.77271E 00	2.30739E-02	9.32084E-02	9.10976E-02	1.54571E 01	3.16583E 02	4	2
5.01188E 03	1.77271E 00	1.41211E-02	1.28191E-01	1.25825E-01	1.54571E 01	5.17393E 02	4	2

Figure 14 - Sample Output Listing - Program 4235

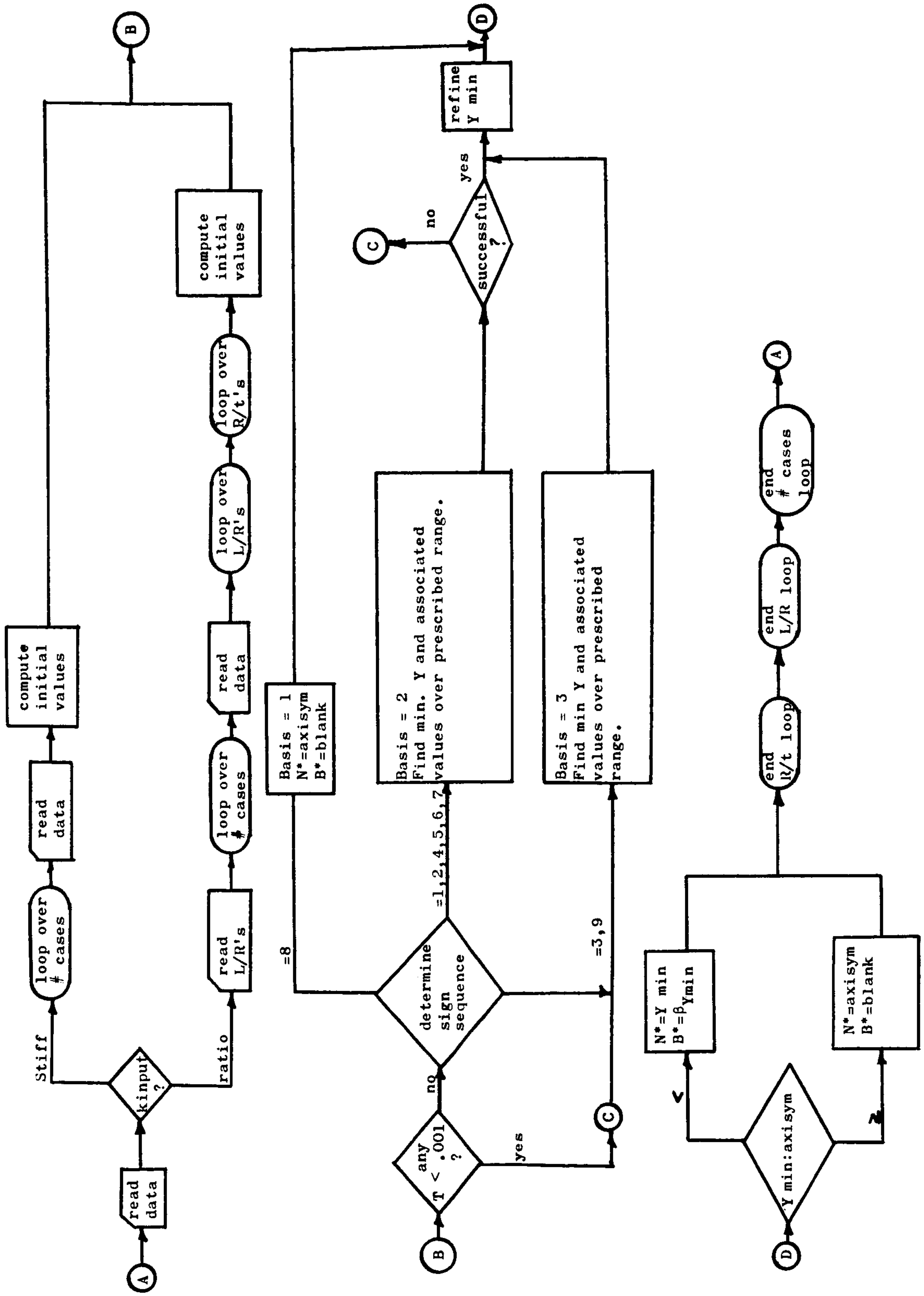

Figure 15 - Flow Diagram - Program 4235

TABLE XIV - Fortran Listing - Program 4235

```
$IBFTC MAIN
      COMMON /BLNK/    BLANK
      COMMON /BOTHOP/ K12,KP,CBETAL,CBETAU,NU,NCASES
      REAL             KP,K12,NU
      COMMON /PRINT/   IBUG
      COMMON /REFINE/ SCREEN,HALFW,NREFIN
      COMMON /RATIO/   PLTMAX,NLR,LR(25),NRT,RT(302),CASENO,KOUTPT,TBART,
     1                 ZBARR
      REAL             LR
      INTEGER          ETAPOP,C12OP
      DATA             BLANK /5HBLANK/
      DATA IRATIO /5HRATIO/, ISTIFF /5HSTIFF/
      CALL SETMIV(125,0,160,176)
      CALL SMXYV(1,0)
   50 WRITE (6,51)
   51 FORMAT (1H1,35X,62HN STAR FOR LONGITUDINALLY STIFFENED CYLINDERS
     *    PROGRAM 4235 )
      CALL TITLE
      READ (5,100) KINPUT,NCASES,NU,CBETAL,CBETAU,PLTMAX,NLR,ETAPOP,
     *                 C12OP,SCREEN,HALFW,NREFIN,IBUG                   MAIN
  100 FORMAT (A5,I5,4F5.4,3I5,2F5.3,2I5)                                MAIN
      IF (ETAPOP.EQ.0 .OR. ETAPOP.EQ.1) GO TO 110
      WRITE (6,105)
  105 FORMAT (5(/),41H0INPUT ERROR -- ETAP OPTION OR C12 OPTION )
      STOP
  110 IF (C12OP.EQ.0 .OR. C12OP.EQ.1) GO TO 120
      WRITE (6,105)
      STOP
  120 KP=ETAPOP
      K12=C12OP
      IF (KINPUT.EQ.IRATIO) KINPUT=1
      IF (KINPUT.EQ.ISTIFF) KINPUT=2
      IF (KINPUT.EQ.1 .OR. KINPUT.EQ.2) GO TO 200
      WRITE (6,150)
  150 FORMAT (/// 54H0INPUT ERROR -- INPUT OPTION IS NOT 'RATIO' OR 'STI
     *FF')
      STOP
  200 IF (KINPUT.NE.1) GO TO 250
      CALL RTCALC (RT,NRT)
      IF (NLR.NE.0) READ (5,205) (LR(I),I=1,NLR)
  205 FORMAT (8E10.5)
      WRITE (6,210) NCASES,NU,CBETAL,CBETAU,PLTMAX,NLR,ETAPOP,C12OP,
     *                 NREFIN,SCREEN,HALFW
  210 FORMAT (5(/),132H INPUT      NUMBER     POISSONS       LOWER
     1UPPER      MAX N STAR         ETA SUB P    C12           SCREENI
     2NG   MIN NO CIRC / 132H OPTION   OF CASES     RATIO      C SUB BETA
     3    C SUB BETA    FOR PLOTS   NLR     OPTION     OPTION    N SUB R
     4 LIMIT      HALF WAVES  / 6H RATIO I9,F12.3,F13.3,F14.3,F12.1,I8,
     5 I9,2I10,F12.3,F13.2)
      IF (NLR.NE.0) WRITE (6,215) (LR(I),I=1,NLR)
  215 FORMAT (// 40X,41HL PRIME/R RATIOS FOR AUTOMATIC SEQUENCING /
```

TABLE XIV - Fortran Listing - Program 4235
(Continued)

```
     *        (1P8E16.5) )
      GO TO 300
  250 WRITE (6,251) NCASES,CBETAL,CBETAU,NREFIN,SCREEN,HALFW
  251 FORMAT (5(/),15X,5HINPUT 11X,6HNUMBER 12X,5HLOWER 11X,5HUPPER 24X,
     1        9HSCREENING 5X,11HMIN NO CIRC / 15X,6HOPTION 9X,8HOF CASES
     2        8X,10HC SUB BETA 6X,10HC SUB BETA 6X,7HN SUB R 11X,5HLIMIT
     35X,14H  HALF WAVES  /15X,5HSTIFF I15,F19.3,F16.3,I12,F19.3,F15.2)
      GO TO 400
C
C     RATIO
  300 CALL RATIOS
      GO TO 50
C
C     STIFF
  400 CALL STIFF
      GO TO 50
      END
$IBFTC TITLE
       SUBROUTINE TITLE
C     SUBROUTINE TO READ CARD WITH TITLE LEFT-ADJUSTED, CENTER TITLE
C
C
      DIMENSION T(61) , BLANK(1)
      DATA  BLANK(1) / 6H      /
      READ (5,101)    (T(J) , J=1,60)
  101 FORMAT (61A1)
      NT=0
      DO 120     J=1,60
      J1= 61-J
      IF ( T (J1).NE. BLANK)  GO TO 150
  120 NT=NT+1
C     NT= NO. BLANKS AFTER TITLE
  150 NT1= 60-NT
      NT2= NT /2
      WRITE (6,160) (BLANK(1), I=1,NT2), (T     (I), I=1,NT1)
  160 FORMAT (1H0 //36X, 61A1)
      RETURN
      END
$IBFTC RTCALD
      SUBROUTINE RTCALC (RT,NRT)
C     SUBROUTINE TO GENERATE 301 VALUES OF R/T FROM 100. TO 10000.
C     EVENLY SPACED RELATIVE TO THE LOG SCALE.
      DIMENSION RT(301)
      NRT=301
      RT(1)=100.
      RT(301)=10000.
      DO 100 I=1,299
      EXP = 2.0 + FLOAT(I)*.00666667
      RT(I+1) = 10. **EXP
  100 CONTINUE
      RT(151)=1000.
```

TABLE XIV - Fortran Listing - Program 4235
(Continued)

```
      RETURN
      END
$IBFTC RATIOD
      SUBROUTINE RATIOS
      COMMON /BLNK/    BLANK
      COMMON /BOTHOP/ K12,KP,CBETAL,CBETAU,NU,NCASES
      COMMON /REFINE/ SCREEN,HALFW,NREFIN
      COMMON /DEBUG/  FC(12),FR(3),FBL,FBU,KDEBUG
      DATA DEBUG /5HDEBUG/
      REAL             KP,K12,NU
      COMMON /MINIMA/ BETAF,NBETA,BETAL,BETAU
      COMMON /PRINT/  IBUG
      COMMON /RATIO/  PLTMAX,NLR,LR(25),NRT,RT(302),CASENO,KOUTPT,TBART,
     1                ZBARR
      REAL             LR
      COMMON /OUT/    BSTAR(302),NSTAR(302),LEFTN(302),RIGHTN(302),
     1                MSTAR(302),AXISYM(302),SINSEQ(302),BASIS(302)
      REAL             NSTAR,LEFTN,MSTAR,LEFTB,KPEPSG
      INTEGER          SINSEQ,BASIS
      DO 1000    NC=1,NCASES
      KDEBUG=0
      READ (5,25) CASENO
   25 FORMAT (A5)
      IF (CASENO-DEBUG) 40,60,40
   40 READ 50, CASENO,KOUTPT,TBART,ZBARR,RT(302),LR(25),BETAF,NBETA
   50 FORMAT (A5,I5,5E10.5,I10)
      FBL =1.
      FBU=1.
      GO TO 90
C
C     TEMPORARY DEBUG INFO
   60 READ 91, (FC(I),I=1,7)
   91 FORMAT (10X,7E10.5)
      READ (5,92) (FC(I),I=8,12),(FR(I),I=1,3)
   92 FORMAT (8E10.5)
      READ (5,93) FBL,FBU
   93 FORMAT (2E10.5)
      KDEBUG=1
   46 READ (5,50) CASENO,KOUTPT,TBART,ZBARR,RT(302),LR(25),BETAF,NBETA
C
C
   90 GO TO (100,100,200,100), KOUTPT
C
C     SETUP TABLE PRINTOUT
  100 WRITE (6,101) CASENO,KOUTPT,TBART,ZBARR,BETAF,NBETA
  101 FORMAT (5(/),25X,6HOUTPUT 56X,4HBETA / 12X, 8HCASE NO. 5X,6HOPTION
     1        11X,7HT BAR/T 15X,7HZ BAR/R 15X,6HFACTOR 14X,10HN SUB BETA
     2        / 12X,A5,I12,1P3E22.5,I17)
      GO TO (300,200,200,310), KOUTPT
  200 CALL PREPLT
C
```

TABLE XIV - Fortran Listing - Program 4235
(Continued)

```
C       COMPUTE VARIABLES NECESSARY FOR MINIMIZATION
C
C       SETUP A LOOP LIKE  NR=NR1,NR2,NR3
C       FOR POINT SOLUTION  NR=302,302,1
C       FOR TABLES          NR=1,256,32  EXCEPT FIRST TIME THRU NR=1,256,31
  300 NL1=1
      NL2=NLR
      GO TO 315
  310 NL1=25
      NL2=25
  315 PI2 = (3.1415926)**2
      STBART = SQRT(TBART)
      KPEPSG = KP/STBART
      ETAS = -NU/STBART + (1.0+NU)*STBART
      DO 600 NL=NL1,NL2
      F = PI2/LR(NL)**2 * STBART * ZBARR
      H = 1.0 + NU*PI2*K12 / LR(NL)**2 * ZBARR
      GO TO (317,317,317,318), KOUTPT
  317 NR=1
      NR2=256
      NR3=31
      GO TO 325
  318 NR=302
      NR2=302
      NR3=1
      GO TO 325
  320 NR3=32
  325 CONTINUE
      BETAL = FBL*(PI2 / (4.0*SQRT(3.0*(1.0- NU**2))*LR(NL)**2*RT(NR)
     *          * SCREEN ) )**.25
      BETAU = FBU*(3.1415926/(.5*HALFW*LR(NL)))*TBART**.25
      ALPHA = (SQRT(3.0*(1.0-NU**2))) /PI2 * LR(NL)**2 * RT(NR) *
     *          (1.0/STBART)
      AXISYM(NR) = ALPHA*H**2/STBART
      IF (IBUG.EQ.0) GO TO 6001
      WRITE (6,6000) BETAL,BETAU
 6000 FORMAT (/// 20H LOWER LIMIT BETA = ,1PE14.7,22H   UPPER LIMIT BETA
     * = ,E14.7)
      WRITE (6,5010) KPEPSG,ETAS,F,H,ALPHA,AXISYM(NR)
 5010 FORMAT (//  8H0KPEPSG=,1PE14.7,8H   ETAS=,E14.7,5H   F=,E14.7 /
     1          3H0H=,E14.7,9H   ALPHA=,E14.7,10H   AXISYM=,E14.7 // )
      WRITE (6,5034) RT(NR)
 5034 FORMAT (//5H0R/T=,1PE14.7)
 6001 CONTINUE
      CALL MIN (ALPHA,ETAS,H,F,KPEPSG,NSTAR(NR),BSTAR(NR),SIMSEQ(NR),
     *          BASIS(NR),AXISYM(NR))
      IF (BSTAR(NR).EQ.BLANK) GO TO 400
      NSTAR(NR) = NSTAR(NR) / STBART
      IF (NSTAR(NR).GT.AXISYM(NR)) GO TO 390
      MSTAR(NR) = 3.1415926/BSTAR(NR)*(1.0/LR(NL))*TBART**.25
      LEFTB = CBETAL*BSTAR(NR)
```

TABLE XIV - Fortran Listing - Program 4235
(Continued)

```
      RIGHTB= CBETAU*BSTAR(NR)
      CALL FUNC (ALPHA,H,F,ETAS,KPEPSG,LEFTB,LEFTN(NR))
      CALL FUNC (ALPHA,H,F,ETAS,KPEPSG,RIGHTB,RIGHTN(NR))
      LEFTN(NR) = LEFTN(NR) / STBART
      RIGHTN(NR) = RIGHTN(NR) / STBART
      GO TO 500
  390 NSTAR(NR)=AXISYM(NR)
      BSTAR(NR)=BLANK
  400 MSTAR(NR) = 0.0
  500 NR=NR+NR3
      IF (NR.LE.NR2) GO TO 320
      CALL OUTPUT (NL)
  600 CONTINUE
 1000 CONTINUE
      RETURN
      END
$IBFTC PREPLD
      SUBROUTINE PREPLI
      COMMON /RATIO/  PLTMAX,NLR,LR(25),NRT,RT(302),CASENO,KOUTPT,TBAPT,
     1                ZBARR
      REAL            LR
C
C     GRID AND TITLES
      IPLT=PLTMAX+.001
      IF (IPLT.LE.0) GO TO 95
      GO TO (100,105,110,115,115,125,130,130,130,130), IPLT
   95 DY=.01
      M=10
      J=-10
      GO TO 150
  100 DY=.02
      M=5
      J=-5
      GO TO 150
  105 DY=.04
      M=5
      J=-5
      GO TO 150
  110 DY=.05
      M=10
      J=-5
      GO TO 150
  115 DY=.1
      M=10
      J=-5
      GO TO 150
  125 DY=.125
      M=8
      J=-4
      GO TO 150
  130 DY=.2
```

TABLE XIV - Fortran Listing - Program 4235 (Continued)

```
      M=5
      J=-5
  150 CALL GRID1V(4,100.,10000.,0.,PLTMAX,1.,DY,1.,M,-1.,J,6,4)
      CALL APRNTV (0,-14,-6,6HN STAR,76,546)
      CALL PRINTV (-10,10HT BAR/T = ,184,876)
      CALL LABLV  (TBART,264,876,-4,1,1)
      CALL PRINTV (-10,10HZ BAR/R = ,847,876)
      IF (ZBARR.EQ.0.) GO TO 200
      CALL LABLV  (ZBARR,927,876,-4,1,1)
      GO TO 210
  200 CALL PRINTV (-2,2H0.,927,876)
  210 CALL PRINTV (-11,11H RADIUS / T,551,120)
  220 CALL RITE2V (333,77,1023,90,1,30,-1,30HMINIMIZATION FACTOR N STAR
     *FOR,NLAST)
      CALL RITE2V (215,45,1023,90,1,43,-1,43HLONGITUDINALLY STIFFENED CI
     *RCULAR CYLINDERS,NLAST)
      RETURN
      END
$IBFTC STIFFD
      SUBROUTINE STIFF
      COMMON /BLNK/    BLANK
      COMMON /BOTHOP/ K12,KP,CBETAL,CBETAU,NU,NCASES
      COMMON /MINIMA/ BETAF,NBETA,BETAL,BETAU
      COMMON /PRINT/  IBUG
      COMMON /REFINE/ SCREEN,HALFW,NREFIN
      REAL            KP,K12,NU
      REAL            NSTAR,MSTAR,LEFTN,LPRIME,LEFTB
      INTEGER         SINSEQ,BASIS
      COMMON /DEBUG/  FC(12),FR(3),FBL,FBU,KDEBUG
      DATA DEBUG /5HDEBUG/
      REAL            KPEP,KPEPSG
      DO 2500 NC=1,NCASES
      KDEBUG=0
      READ (5,25) CASENO
   25 FORMAT (A5)
      IF (CASENO-DEBUG) 40,90,40
   40 READ 100,CASENO,A11,A22,A12,A33,C11,R,BETAF
  100 FORMAT (5X,A5,7E10.5)
      FBL =1.
      FBU=1.
      GO TO 101
C
C     TEMPORARY DEBUG INFO
   90 READ 91, (FC(I),I=1,7)
   91 FORMAT (10X,7E10.5)
      READ (5,92) (FC(I),I=8,12),(FR(I),I=1,3)
   92 FORMAT (8E10.5)
      READ (5,93) FBL,FBU
   93 FORMAT (2E10.5)
      KDEBUG=1
   99 READ (5,100) CASENO,A11,A22,A12,A33,C11,R,BETAF
```

TABLE XIV - Fortran Listing - Program 4235
(Continued)

```
C
C
  101 READ (5,102) D11,D22,D12,D33,C12,LPRIME,NBETA
  102 FORMAT (10X,6E10.5,I10)
      PI2=(3.1415926)**2
      KPEP = KP*(D12 +2.0*D33) / SQRT(D11*D22)
      GAMMA= D11*A11/(D22*A22)
      KPEPSG = KPEP * SQRT(GAMMA)
      ETAS = (A12 + A33/2.0) / SQRT(A11*A22)
      SD22A1=SQRT(D22/A11)
      ALPHA = LPRIME**2/(2.0*PI2*R*A22*SD22A1)
      F = C11 / (2.0*ALPHA*SQRT(A22*D22))
      H = 1.0 - (K12*C12 / (2.0*ALPHA*A22*SD22A1))
      AXISYM = ALPHA*H**2
      BETAL = FBL*(1.0/(4.0*ALPHA*SCREEN))**.25
      BETAU = FBU * (3.1415926*R /(.5*HALFW*LPRIME)) * (A22/A11)**.25
      IF (IBUG.EQ.0) GO TO 6001
      WRITE (6,6000) BETAL,BETAU
 6000 FORMAT (/// 20H LOWER LIMIT BETA = ,1PE14.7,22H   UPPER LIMIT BETA
     * = ,E14.7)
      WRITE (6,5000) KPEPSG,ETAS,SD22A1,ALPHA,F,H,AXISYM
 5000 FORMAT (// 8H0KPEPSG=,1PE14.7,8H   ETAS=,E14.7,10H   SD22A1=,
     1       E14.7,9H   ALPHA=,E14.7 / 3H0F=,E14.7,5H   H=,E14.7,
     2       10H   AXISYM=,E14.7 )
 6001 CONTINUE
      CALL MIN (ALPHA,ETAS,H,F,KPEPSG,NSTAR,BSTAR,SINSEQ,BASIS,AXISYM)
      IF (BSTAR.EQ.BLANK) GO TO 190
      IF (NSTAR.LE.AXISYM) GO TO 200
      NSTAR=AXISYM
      BSTAR=BLANK
  190 MSTAR=0.0
      GO TO 2000
  200 MSTAR = (3.1415926*R)/(BSTAR*LPRIME) * (A22/A11)**.25
      LEFTB = CBETAL*BSTAR
      RIGHTB= CBETAU*BSTAR
      CALL FUNC (ALPHA,H,F,ETAS,KPEPSG,LEFTB,LEFTN)
      CALL FUNC (ALPHA,H,F,ETAS,KPEPSG,RIGHTB,RIGHTN)
C
C     PRINT RESULTS
 2000 WRITE (6,2001) CASENO,A11,A22,A12,A33,C11,R,BETAF
 2001 FORMAT (5(/),119X,4HBETA / 3X,8HCASE NO. 7X,3HA11 14X,3HA22 14X,
     1       3HA12 14X,3HA33 14X,3HC11 15X,1HR 13X,6HFACTOR / 3X,A5,
     2       1P7E17.5 )
      WRITE (6,2003) D11,D22,D12,D33,C12,LPRIME,NBETA
 2003 FORMAT (// 18X,3HD11 14X,3HD22 14X,3HD12 14X,3HD33 14X, 3HC12 12X,
     1       7HL PRIME 9X,10HN SUB BETA / 8X,1P6E17.5,I14)
      WRITE (6,2005)
 2005 FORMAT (// 33X,7HMINIMUM 11X,4HLEFT 13X,5HRIGHT10X,8HCRITICAL 10X,
     1       6HAXISYM 8X,4HSIGN / 15X,9HBETA STAR 10X,5HVALUE 12X,
     2       5HVALUE 12X,5HVALUE 8X,12HLOWER CASE N 9X,5HVALUE 6X,
     3       8HSEQUENCE 2X, 5HBASIS)
```

TABLE XIV - Fortran Listing - Program 4235
(Continued)

```
      IF (BSTAR.EQ.BLANK) GO TO 2100
      WRITE (6,2010) BSTAR,NSTAR,LEFTN,RIGHTN,MSTAR,AXISYM,SINSEQ,
     *               BASIS
 2010 FORMAT (8X,1P6E17.5,2I8)
      GO TO 3000
 2100 WRITE (6,2101) AXISYM,MSTAR,AXISYM,SINSEQ,BASIS
 2101 FORMAT (25X,1PE17.5,34X,2E17.5,2I8)
 2500 CONTINUE
 3000 RETURN
      END
$IBFTC MIND
      SUBROUTINE MIN (ALPHA,ETAS,H,F,KPEPSG,NSTAR,BSTAR,SINSEQ,BASIS,
     *                AXISYM)
      COMMON /MINIMA/ BETAF,NBETA,BETAL,BETAU
      COMMON /PRINT/  IBUG
      DIMENSION       T(3),C(12)
      COMMON /BLNK/   BLANK
      COMMON /DEBUG/  FC(12),FR(3),FBL,FBU,KDEBUG
      REAL            KPEP,KPEPSG
      REAL            NSTAR
      INTEGER         SINSEQ,BASIS
C
C     TEST FOR MINIMIZATION PROCEDURE TO EMPLOY
      TWOALF = 2.0*ALPHA
      C(1) = TWOALF*ETAS*H**2
      C(2) = TWOALF*H*F
      C(3) = -KPEPSG/TWOALF
      IF (KDEBUG.EQ.0) GO TO 2
      DO 1 I=1,3
    1 C(I) = C(I)*FC(I)
    2 CONTINUE
      CNEG=0.0
      CPOS=0.0
      DO 105 I=1,3
      IF (C(I)) 102,105,104
  102 CNEG=CNEG+C(I)
      GO TO 105
  104 CPOS=CPOS+C(I)
  105 CONTINUE
      T(1)=ABS(ABS(CNEG/CPOS) - 1.0)
      C(4) = TWOALF*H**2
      C(5) = -TWOALF*F**2
      C(6) = -2.0*ETAS*KPEPSG/ALPHA
      C(7) = -1.0/TWOALF
      IF (KDEBUG.EQ.0) GO TO 4
      DO 3 I=4,7
    3 C(I) = C(I)*FC(I)
    4 CONTINUE
      CNEG=0.0
      CPOS=0.0
      DO 115 I=4,7
```

TABLE XIV - Fortran Listing - Program 4235
(Continued)

```
      IF (C(I)) 112,115,114
  112 CNEG=CNEG+C(I)
      GO TO 115
  114 CPOS=CPOS+C(I)
  115 CONTINUE
      T(2)=ABS(ABS(CNEG/CPOS) - 1.0)
      C(8) = -TWOALF*H*F
      C(9) = -TWOALF*ETAS*F**2
      C(10)= -KPEPSG/ALPHA
      C(11)= -2.0*ETAS**2*KPEPSG / ALPHA
      C(12)= -2.0*ETAS/ALPHA
      IF (KDEBUG.EQ.0) GO TO 6
      DO 5 I=8,12
    5 C(I) = C(I)*FC(I)
    6 CONTINUE
      CNEG=0.0
      CPOS=0.0
      DO 125 I=8,12
      IF (C(I)) 122,125,124
  122 CNEG=CNEG+C(I)
      GO TO 125
  124 CPOS=CPOS+C(I)
  125 CONTINUE
      T(3)=ABS(ABS(CNEG/CPOS) - 1.0)
      IF (IBUG.NE.0) WRITE (6,5001) T
 5001 FORMAT (// 6H0T(1)=,1PE20.7,8H    T(2)=,E20.7,8H    T(3)=,E20.7)
      SINSEQ=0
      DO 130 I=1,3
      IF (T(I).LT. .001) GO TO 600
  130 CONTINUE
C
C     DETERMINE SIGN SEQUENCE
      A1 = C(1)+C(2)+C(3)
      A2 = C(4)+C(5)+C(6)+C(7)
      A3 = C(8)+C(9)+C(10)+C(11)+C(12)
      IF (IBUG.EQ.0) GO TO 5013
      WRITE (6,5011) (C(I),I=1,12)
 5011 FORMAT (// 7X,1HC / (1PE15.7))
      WRITE (6,5012) A1,A2,A3
 5012 FORMAT (//4H0A1=,1PE14.7,6H    A2=,E14.7,6H    A3=,E14.7)
 5013 CONTINUE
      IF (A1) 211,230,200
  200 IF (A2) 206,230,201
  201 IF (A3) 204,230,202
  202 SINSEQ=1
      GO TO 300
  204 SINSEQ=2
      GO TO 300
  206 IF (A3) 209,230,207
  207 SINSEQ=3
      GO TO 300
```

TABLE XIV - Fortran Listing - Program 4235
(Continued)

```
  209 SINSEQ=4
      GO TO 300
  211 IF (A2) 217,230,212
  212 IF (A3) 215,230,213
  213 SINSEQ=5
      GO TO 300
  215 SINSEQ=6
      GO TO 300
  217 IF (A3) 220,230,218
  218 SINSEQ=7
      GO TO 300
  220 SINSEQ=8
      GO TO 300
  230 SINSEQ=9
  300 CONTINUE
      IF (IBUG.EQ.0) GO TO 5003
      WRITE (6,5002) SINSEQ
 5002 FORMAT (//8H0SINSEQ=,I2)
 5003 CONTINUE
      GO TO (500,500,600,500,500,500,500,400,600), SINSEQ
C
C     CASE A (MINIMUM IS ASYMPTOTE)
  400 BASIS = 1
      NSTAR = AXISYM
      BSTAR = BLANK
      GO TO 1100
C
C     CASE B
  500 BASIS = 2
      CALL CASEB (ALPHA,H,F,ETAS,KPEPSG,BSTAR,NSTAR,IR)
      IF (IBUG.EQ.0) GO TO 5034
      WRITE (6,5033) IR
 5033 FORMAT (//6H0IR = ,I2)
 5034 CONTINUE
      IF (IR) 600,1000,600
C
C     CASE C
  600 BASIS = 3
      CALL CASEC (ALPHA,H,F,ETAS,KPEPSG,BSTAR,NSTAR)
 1000 CALL SEARCH (ALPHA,H,F,ETAS,KPEPSG,BSTAR,NSTAR,BETAF,BETAU)
 1100 RETURN
      END
$IBFTC CASEBD
      SUBROUTINE CASEB (ALPHA,H,F,ETAS,KPEPSG,BSTAR,NSTAR,IR)
      COMMON /DEBUG/  FC(12),FR(3),FBL,FBU,KDEBUG
      COMMON /PRINT/  IBUG
      COMMON /MINIMA/ BETAF,NBETA,BETAL.BETAU
      REAL            NSTAR,KPEPSG
      DIMENSION       BETA(3),Y(3)
      IR=0
```

TABLE XIV - Fortran Listing - Program 4235
(Continued)

```
      TEMP = BETAU/BETAF**5
      IF (IBUG.NE.0) WRITE (6,7000) TEMP
 7000 FORMAT (/// 38H UPPER LIMIT BETA / BETA FACTOR **5 = ,1PE14.7)
      IF (.1 - TEMP) 50,50,60
   50 BETA(1) = .1
      BETA(2) = .1
      BETA(3) = .1
      GO TO 70
   60 BETA(1) = TEMP
      BETA(2) = TEMP
      BETA(3) = TEMP
   70 CALL FUNC (ALPHA,H,F,ETAS,KPEPSG,BETA(1),Y(1))
      IF (IBUG.EQ.0) GO TO 5005
      WRITE (6,5003)
 5003 FORMAT (//24H0SUB. CASEB DEBUG VALUES /11X,4HBETA 18X,1HY)
      WRITE (6,5004) BETA(1),Y(1)
 5004 FORMAT (1P2E20.7)
 5005 CONTINUE
      Y(2) = Y(1)
      DO 100 I=2,NBETA
      BETA(3)=BETAF*BETA(3)
      CALL FUNC (ALPHA,H,F,ETAS,KPEPSG,BETA(3),Y(3))
      IF (IBUG.NE.0) WRITE (6,5004) BETA(3),Y(3)
      IF (Y(3).GE.Y(2)) GO TO 90
      Y(2)=Y(3)
      BETA(2)=BETA(3)
   90 IF (BETA(3).GT.BETAU) GO TO 110
  100 CONTINUE
  110 CONTINUE
C
C     Y(1), BETA(1) -- INITIAL VALUES
C     Y(2), BETA(2) -- MINIMUM VALUES
C     Y(3), BETA(3) -- FINAL   VALUES
C
C     CALCULATE R1, R2, AND R3
C
C
      R1 = ALPHA*H**2/Y(2)
      IF (KDEBUG.NE.0) R1=R1*FR(1)
      IF (IBUG.NE.0) WRITE (6,5030) R1
 5030 FORMAT (//5H0R1= ,1PE14.7)
      IF (R1.GE. 1.001) GO TO 200
      IR=1
      GO TO 300
  200 R2 = Y(1)/Y(2)
      IF (KDEBUG.NE.0) R2=R2*FR(2)
      IF (IBUG.NE.0) WRITE (6,5031) R2
 5031 FORMAT (//5H0R2= ,1PE14.7)
      IF (R2.GE. 1.001) GO TO 210
      IR=1
      GO TO 300
```

TABLE XIV - Fortran Listing - Program 4235
(Continued)

```
  210 R3 = Y(3)/Y(2)
      IF (KDEBUG.NE.0) R3=R3*FR(3)
      IF (IBUG.NE.0) WRITE (6,5032) R3
 5032 FORMAT (//5H0R3= ,1PE14.7)
      IF (R3.GE. 1.001) GO TO 220
      IR=1
      GO TO 300
  220 BSTAR=BETA(2)
      NSTAR=Y(2)
  300 RETURN
      END
$IBFTC CASECD
      SUBROUTINE CASEC (ALPHA,H,F,ETAS,KPEPSG,BSTAR,NSTAR)
      COMMON /MINIMA/ BETAF,NBETA,BETAL,BETAU
      COMMON /PRINT/  IBUG
      REAL            NSTAR,MSTAR,LEFTN,LPRIME
      REAL            KPEP,KPEPSG
      TEMP = BETAU/BETAF**5
      IF (IBUG.EQ.0) GO TO 7001
      WRITE (6,7000) TEMP
 7000 FORMAT (/// 38H UPPER LIMIT BETA / BETA FACTOR **5 = ,1PE14.7)
      WRITE (6,5005)
 5005 FORMAT (//24H0SUB. CASEC DEBUG VALUES /11X,4HBETA,18X,1HY )
 7001 CONTINUE
      IF (BETAL - TEMP) 50,50,60
   50 BETA = BETAL
      BMIN = BETAL
      GO TO 70
   60 BETA = TEMP
      BMIN = TEMP
   70 CALL FUNC (ALPHA,H,F,ETAS,KPEPSG,BMIN,YMIN)
      IF(IBUG.NE.0) WRITE (6,5006) BMIN,YMIN
  100 BETA = BETA * BETAF
      CALL FUNC (ALPHA,H,F,ETAS,KPEPSG,BETA,Y)
      IF (IBUG.NE.0) WRITE (6,5006) BETA,Y
 5006 FORMAT (1P2E20.7)
      IF (Y.GT.YMIN) GO TO 180
      YMIN=Y
      BMIN=BETA
  180 IF (BETA.LE.BETAU) GO TO 100
      NSTAR = YMIN
      BSTAR = BMIN
      RETURN
      END
$IBFTC SEARCD
      SUBROUTINE SEARCH (ALPHA,H,F,ETAS,KPEPSG,BSTAR,NSTAR,BETAF,BETAU)
      COMMON /PRINT/  IBUG
      DIMENSION       BETA(9),Y(9)
      COMMON /REFINE/ SCREEN,HALFW,NREFIN
      REAL NSTAR
      IF (NREFIN.LE.0) RETURN
```

TABLE XIV - Fortran Listing - Program 4235
(Continued)

```
      N=1
      BF = BETAF
      BMIN = BSTAR
   50 BF = SQRT(BF)
      BETA(5) = BMIN
      DO 60 I=1,4
      NEXP = 5-I
      BETA(I) = BETA(5)/BF**NEXP
      BETA(I+5) = BETA(5)*BF**I
   60 CONTINUE
      DO 80 I=1,9
   80 IF (BETA(I).GT.BETAU) BETA(I)=BETAU
  100 YMIN=1.0E+38
      DO 110 I=1,9
      CALL FUNC (ALPHA,H,F,ETAS,KPEPSG,BETA(I),Y(I))
      IF (Y(I).GE.YMIN) GO TO 110
      YMIN=Y(I)
      BMIN=BETA(I)
      IMIN=I
  110 CONTINUE
      IF (IBUG.EQ.0) GO TO 1002
      WRITE (6,1000) N,(BETA(J),Y(J),J=1,9)
 1000 FORMAT (/// 26H SUB. SEARCH -- ITERATION I3 / 11X,4HBETA 18X,1HY /
     *           (1P2E20.7))
      WRITE (6,1001) YMIN,BMIN,IMIN
 1001 FORMAT (// 9H Y MIN = ,1PE14.7,14H   BETA MIN = ,1PE14.7,
     *           11H   I MIN = ,I1)
 1002 CONTINUE
      N=N+1
      IF (N.LE.NREFIN) GO TO 50
  300 NSTAR = YMIN
      BSTAR = BMIN
      RETURN
      END
$IBFTC PLOTD
      SUBROUTINE PLOT
      COMMON /RATIO/  PLTMAX,NLR,LR(25),NRT,RT(302),CASENO,KOUTPT,TBAPT,
     1                ZBARR
      REAL            LR
      COMMON /OUT/    BSTAR(302),NSTAR(302),LEFTN(302),RIGHTN(302),
     1                MSTAR(302),AXISYM(302),SINSEQ(302),BASIS(302)
      REAL            NSTAR,LEFTN,MSTAR
      INTEGER         SINSEQ,BASIS
C
C     PLOT ACTUAL CALCULATED POINTS OF N STAR VS. R/T
      CALL APLOTV (256,RT(32),NSTAR(32),32,32,1,1HX,IERR)
      CALL POINTV (RT(1),NSTAR(1),2)
      RETURN
      END
$IBFTC FUNCD
      SUBROUTINE FUNC (ALPHA,H,F,ETAS,KPEPSG,BETA,Y)
```

TABLE XIV - Fortran Listing - Program 4235
(Continued)

```
      REAL KPEPSG
      Y = KPEPSG/(2.0*ALPHA*BETA**2) + 1.0/(4.0*ALPHA*BETA**4) +
     1        (ALPHA*BETA**4*(H-F/BETA**2)**2) / (1.0+2.0*ETAS*BETA**2
     2        +BETA**4)
      RETURN
      END
$IBFTC OUTPUD
      SUBROUTINE OUTPUT (NL)
      COMMON /RATIO/  PLTMAX,NLR,LR(25),NRT,RT(302),CASENO,KOUTPT,TBART,
     1                ZBARR
      REAL            LR
      COMMON /BLNK/   BLANK
      COMMON /OUT/    BSTAR(302),NSTAR(302),LEFTN(302),RIGHTN(302),
     1                MSTAR(302),AXISYM(302),SINSEQ(302),BASIS(302)
      REAL            NSTAR,LEFTN,MSTAR
      INTEGER         SINSEQ,BASIS
C
 1000 GO TO (1050,1050,1200,1500), KOUTPT
C
C     TABLES
 1050 WRITE (6,1051) LR(NL)
 1051 FORMAT (///60X,9HL PRIME/R /54X,1PE16.5// 86X,8HCRITICAL 12X,1HN
     1       10X,4HSIGN/9X,3HR/T 10X,9HBETA STAR 8X,6HN STAR 10X,
     2       6HLEFT N 10X,7HRIGHT N 6X,12HLOWER CASE N 7X,6HAXISYM 6X,
     3       8HSEQUENCE 2X,5HBASIS)
      NR=1
      NR2=256
      NR3=31
      GO TO 1102
 1101 NR3=32
 1102 CONTINUE
      IF (BSTAR(NR).EQ.BLANK) GO TO 1110
      WRITE (6,1103) RT(NR),BSTAR(NR),NSTAR(NR),LEFTN(NR),RIGHTN(NR),
     1                MSTAR(NR),AXISYM(NR),SINSEQ(NR),BASIS(NR)
 1103 FORMAT (1P7E16.5,2I8)
      GO TO 1125
 1110 WRITE (6,1111) RT(NR), NSTAR(NR),MSTAR(NR),AXISYM(NR),SINSEQ(NR),
     *                BASIS(NR)
 1111 FORMAT (1PE16.5,16X,E16.5,32X,2E16.5,2I8)
 1125 NR=NR+NR3
      IF (NR.LE.NR2) GO TO 1101
      GO TO (3000,1200,1200,1500), KOUTPT
C
C     PLOTS
 1200 CALL PLOT
      GO TO 3000
C
C     POINT SOLUTION
 1500 WRITE (6,1051) LR(25)
      IF (BSTAR(302).EQ.BLANK) GO TO 1510
      WRITE (6,1103) RT(302),BSTAR(302),NSTAR(302),LEFTN(302),
```

TABLE XIV - Fortran Listing - Program 4235
(Continued)

```
     1                   RIGHTN(302),MSTAR(302),AXISYM(302),SINSEQ(302),
     2                   BASIS(302)
      GO TO 3000
 1510 WRITE (6,1111) RT(302),NSTAR(302),MSTAR(302),AXISYM(302),
     *                   SINSEQ(302),BASIS(302)
 3000 RETURN
      END
```

SECTION 8

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